FLOATING ZONE-EQUILIBRIUM SHAPES AND STABILITY CRITERIA

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For the determination of the shape of a floating liquid zone under hydrostatic equilibrium conditions (no relative velocity field in the liquid), we can take into account the effect of any non-time-dependant acceleration of the reference frame. The resulting shapes can be axisymmetric and non-axisymmetric. According to Fig. 1,

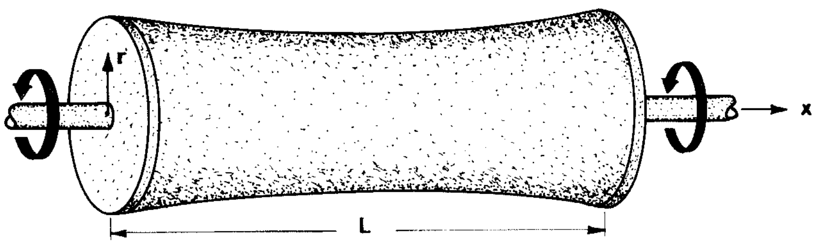


Fig. 1. Floating liquid zone.

axisymmetric equilibrium shapes are possible even with an axial constant acceleration and rigid body rotation about the axis. Non-axisymmetric equilibrium shapes appear in non-rotating conditions with a non-axial constant component of the acceleration, and they are not studied here. For a given set of boundary conditions, the solution is not single-valued, and the numerical computations must follow a parametric differentiation procedure [1]. However, in the absence of gravity and without rotation, the problem admits an analytical solution in a comprehensive and closed way, very useful for a later step devoted to the stability criterion study; in this solution, the free surface equation becomes

$$x = F(\frac{\pi}{2},\alpha) - F(\varphi,\alpha) + E(\frac{\pi}{2},\alpha) - E(\varphi,\alpha) \;/\cos\alpha$$

when F and E are the elliptic integrals of the first and second class, α a parameter connected to the geometry of the zone, and _ a variable related to the radius by $r^2 = (1-\sin^2\alpha \sin^2\varphi)/\cos^2\alpha$. The origin of the x axis is placed midway between the discs, where a plane of symmetry exists.

The principal results in the axisymmetric shape study concern the minimum volumen of liquid which can be placed between the discs, and are summarized in Fig. 2.

Although the analysis of those equilibrium shapes is very important for the management of real liquid zones in the experiments [2], and referring to previous work on the stability of the zone [3], the most interesting shape from

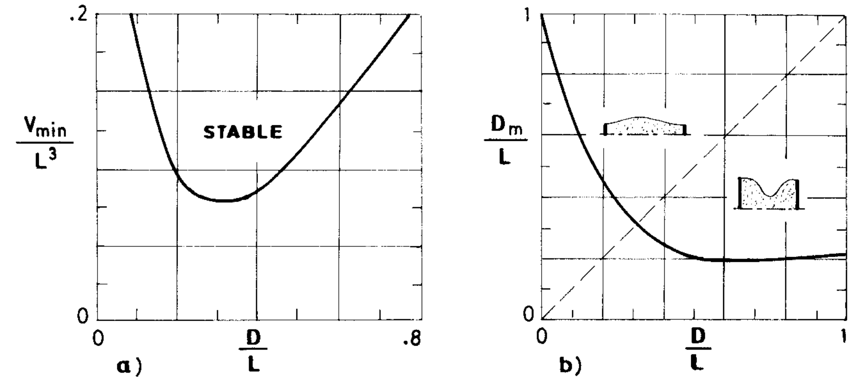


Fig. 2. Dimensionless minimum stable volume between discs of diameter D and separation L. b) Dimensionless minimum diameter at midway between the discs.

the applications point of view (and from the simplicity one), is doubtless the cylindrical shape.

The cylindrical shape is a possible equilibrium configuration for a liquid zone under any spin rate in the absence of residual acceleration, but, will this equilibrium become unstable for large values of the parameters? The answer has been known for a long time [4] and is that it will. We have performed a detailed linear stability analysis for a cylindrical liquid zone regarding axisymmetric and non-axisymmetric perturbations, based on the variational principle [5]. Several contact conditions at the discs have been considered: anchored edges, free edges and mixed conditions though the reality of the free edge configuration is in doubt because of its inherent instability. The results are synthesized in Fig. 3.

All the previously mentioned analyses were concerned with the hydrostatic equilibrium, the first task in a new fluid dynamics problem. A second task should be to deal with motions of the free surface, which are always easier to understand and the easiest to record. Later on, we shall study the internal flow in the zone.

From the experimental point of view, one of the primary items in the behaviour of a floating liquid zone seems to be its response to vibrations, which are always present in a real case. With this aim, we have done a linear steady state analysis for a cylindrical zone subject to forced harmonic oscillations in the axial direction. The linearity of the process is based on the inequality $\delta f^2 \rho L^3/\sigma \ll 1$, where δ and f are the amplitude and frequency of the vibration, ρ the density of the liquid, L the length of the zone, and σ the surface tension. Unfortunately, this linear approach does not allow us to take into account all the three possible different contact conditions but only the free edges one, as in Fig. 4, where the maximum amplitude of the deformation is plotted. In spite of this fact, it seems possible to extrapolate these results (Fig. 5) to the other possible contact conditions in such a way as to preserve the well known stability limits for $f = 0$.

Although it has not been included in this summary, the analysis when the zone

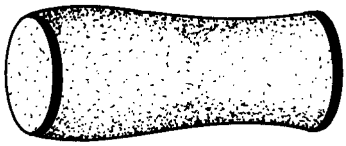
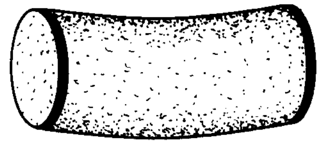
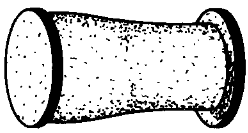
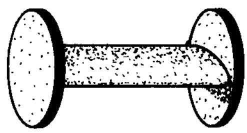
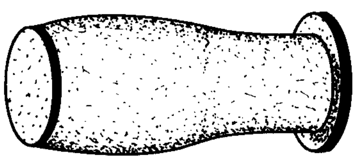
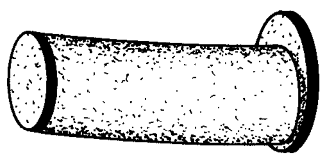
Modes with anchored edges		
	amphora shape	"C" shape
Maximum stable length	$L_{max}=2\pi R\left(1+\frac{\rho\Omega^2R^3}{\sigma}\right)^{-1/2}$	$L_{max}=\frac{\pi}{\Omega}\sqrt{\frac{\sigma}{\rho R}}$
valid for	$\Omega\leq\sqrt{\frac{\sigma}{3\rho R^3}}$	$\Omega\geq\sqrt{\frac{\sigma}{3\rho R^3}}$
Maximum allowable Ω	$\Omega_{max}=\sqrt{\frac{\sigma}{\rho R^3}\left[\left(2\pi\frac{R}{L}\right)^2-1\right]}$	$\Omega_{max}=\frac{\pi}{L}\sqrt{\frac{\sigma}{\rho R}}$
valid for	$L\geq\sqrt{3}\,\pi R$	$L\leq\sqrt{3}\,\pi R$
Modes with free edges		
	bottle shape	elliptic cylinder shape
Maximum stable length	$L_{max}=\pi R\left(1+\frac{\rho\Omega^2R^3}{\sigma}\right)^{-1/2}$	independent of L
valid for	$\Omega\leq\sqrt{\frac{\sigma}{3\rho R^3}}$	$\Omega\geq\sqrt{\frac{\sigma}{3\rho R^3}}$
Modes with one edge anchored and the other free		
	amphora shape	semi "C" shape
Maximum stable length	$L_{max}=1.43\,\pi R\left(1+\frac{\rho\Omega^2R^3}{\sigma}\right)^{-1/2}$	$L_{max}=\frac{\pi}{2\Omega}\sqrt{\frac{\sigma}{\rho R}}$
valid for	$\Omega\leq\sqrt{\frac{\sigma}{7.2\,\rho\,R^3}}$	$\Omega\geq\sqrt{\frac{\sigma}{7.2\,\rho\,R^3}}$
Maximum allowable Ω	$\Omega_{max}=\sqrt{\frac{\sigma}{\rho R^3}\left[\left(1.43\,\pi\frac{R}{L}\right)^2-1\right]}$	$\Omega_{max}=\frac{\pi}{2L}\sqrt{\frac{\sigma}{\rho R}}$
valid for	$L\geq 1.33\,\pi R$	$L\leq 1.33\,\pi R$

Fig. 3. Stability limits for a cylindrical liquid zone of radius R and length L, rotating at angular speed Ω; ρ and σ are the density and the surface tension respectively.

is rotating as a whole is very similar, provided that the rotation rate does not exced π times the vibration frequency [6].

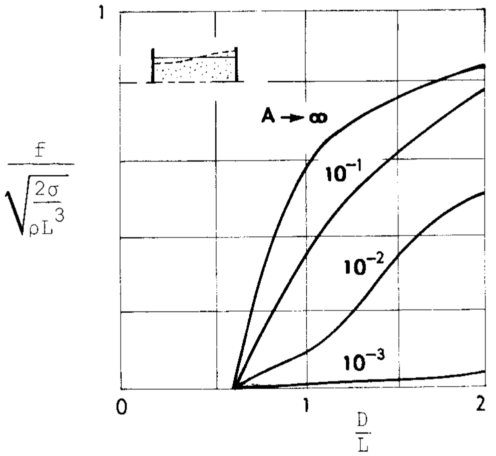


Fig. 4. Stability limit and maximum distortion proportional amplitude of a cylindrical zone subject to an axial vibration of frequency f.

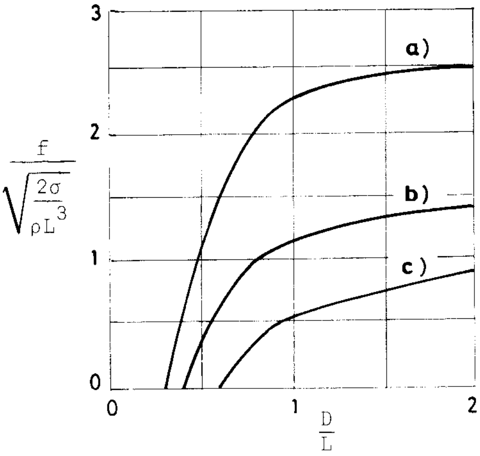


Fig. 5. Estimated stability limits for a cylindrical zone:
a) anchored edges
b) free edges
c) mixed conditions

Acknowledgement

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References

1. J.F. Padday, J. Electroanalytical and Interf. Electrochem. 37, 313 (1972).
2. J.M. Ruesga, CASA. ESA Contract SP/HQ/95, 1976.
3. J.F. Padday, in: Material Sciences in Space, ESA SP-114, 1976, p. 477.
4. Lord Rayleigh, The Theory of Sound, II. Dover Publ. 1945, p. 351.
5. I. Martínez, in: Material Sciences in Space, ESA SP-114, 1976, p. 247.
6. H.P. Greenspan, The Theory of Rotating Fluids, Cambridge Univ. Press 1968, p. 10.