

Statement

In order to produce 100 kg/h of liquid nitrogen from the gas at room conditions, a cascade of 5 compressors (with inter-cooling to the environment) is used to get the gas at 15 MPa, with compressor isentropic efficiencies of 86%, followed by the liquefier itself, where the gas goes along a coil and discharges through a valve to atmospheric pressure. Evaluate:

- Minimum required power (perfect path).
- Optimum intermediate pressures.
- Fraction of the gas stream that is liquefied.
- Demanded power and exergy efficiency of the plant.

Se desea obtener 100 kg/hora de nitrógeno líquido partiendo del gas en condiciones ambientes. Para ello se va a utilizar una cascada de 5 compresores que elevarán la presión hasta 15 MPa con rendimiento adiabático 0,86 (con enfriamiento intermedio hasta la atmósfera), y el licuador propiamente dicho, donde el nitrógeno pasa por un serpentín y descarga en una válvula hasta la presión atmosférica. Se pide:

- Potencia mínima necesaria (camino perfecto).
- Presiones intermedias óptimas.
- Fracción de gasto licuado.
- Potencia necesaria y rendimiento exergético de la planta.

Solution

- Minimum required power (perfect path).

The liquid should be obtained at ambient pressure, to avoid the need of thick-wall containers on this cryogenic installation. The minimum work required to pass nitrogen from ambient conditions to cryogenic liquid at its normal boiling point is:

$$\dot{W}_{u,\min} = \dot{m}\Delta\psi = \dot{m}(\Delta h_t - T_0\Delta s) \rightarrow \begin{cases} \Delta h = -c_p(T_0 - T_b) - h_{lv_b} = -1(288 - 77) - 199 = -410 \text{ kJ/kg} \\ \Delta s = -c_p \ln \frac{T_0}{T_b} - \frac{h_{lv_b}}{T_b} = -1 \ln \frac{288}{77} - \frac{199}{77} = -3.9 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \end{cases}$$

i.e. $\dot{W}_{u,\min} = 19.8 \text{ kW}$ for the $\dot{m}_{liq} = 100/3600 = 0.028 \text{ kg/s}$ of liquid produced, where the perfect gas model has been assumed up to cryogenic temperatures, with the room temperature thermal-capacity-value. Most accurate data from the nitrogen p - h diagram below, gives $\dot{W}_{u,\min} = \dot{m}_w[(h_0 - h_b) - T_0(s_0 - s_b)] = (100/3600)[(300 + 120) - 288(6.8 - 2.9)] = 19.5 \text{ kW}$.

The theoretical path corresponding to this thermodynamic limit, the perfect path, would consist of an isothermal compression, horizontally to the left from point A in Fig. 1 (where the classical

Linde gas-liquefaction process is presented) up to the pressure of point corresponding to crossing with the isentropic from D1 (vertical from D1 in Fig. 1).

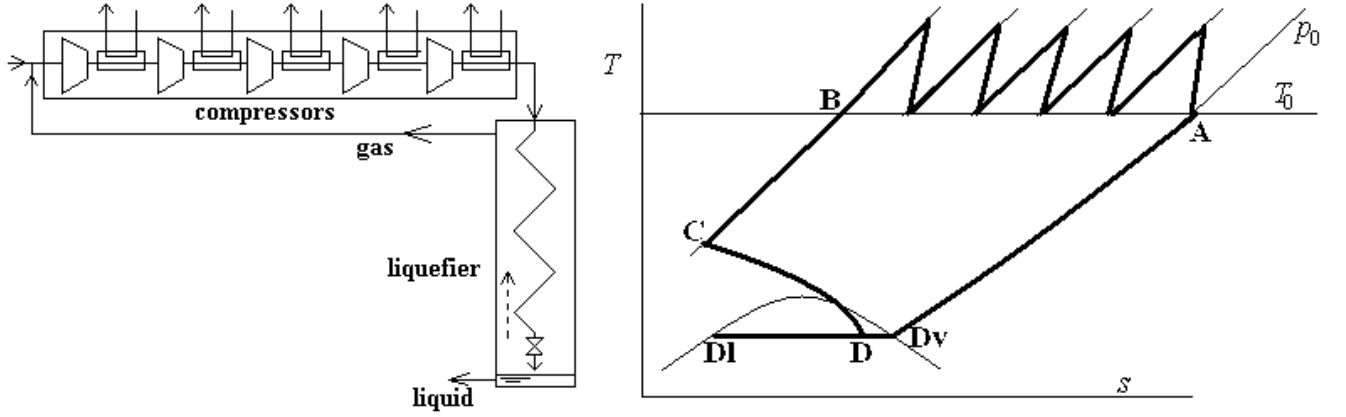


Fig. 1. Sketch of the components and the T - s diagram of the processes (Linde liquefaction).

b) Optimum intermediate pressures.

The intermediate pressure values (p_1, p_2, p_3, p_4) that minimise global compressor power needed, may be guessed with the perfect gas model, which has a simple analytical solution; otherwise, a multi-parametric trial and error optimisation is involved. Thence:

$$w_{tot} = \sum_{i=1}^5 c_p (T_i - T_0) = \sum_{i=1}^5 c_p T_0 \left[\frac{\left(\frac{p_i}{p_{i-1}} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \right] \rightarrow \frac{\partial w_{tot}}{\partial p_i} = 0 \Rightarrow \frac{p_1}{p_0} = \frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \frac{p_5}{p_4}$$

and finally:

$$p_i = (p_0^{5-i} p_5^i)^{1/5} \longrightarrow p_i = \{100, 270, 740, 2000, 5500, 15000\} \text{ kPa}$$

i.e., with the perfect-gas model, each compressor is equally loaded, with the same pressure ratio and same exit temperature (in the limit of inter-cooling down to ambient temperature, an idealisation, of course, because it would require an infinite heat-exchange area). Figure 1 presents the components and processes involved.

Fixing the above pressure values, but using the real p - h diagram for N_2 , the multistage compression process can be worked out as usual (first the isentropic compression is depicted, and then the real final-enthalpy computed from the isentropic-efficiency definition), and is shown in Fig. 2.

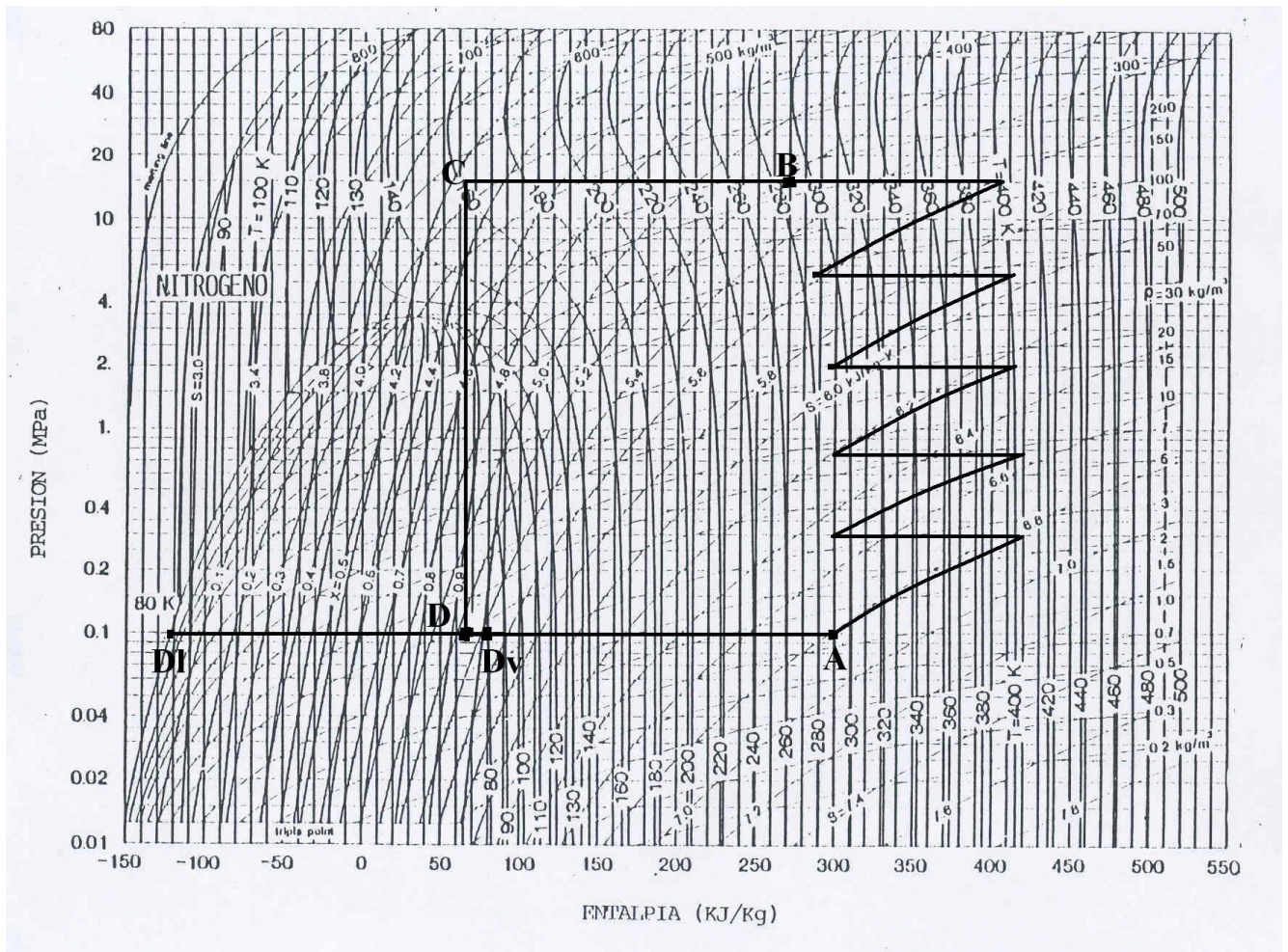


Fig. 2. Sketch of the processes in p - h diagram for nitrogen.

- c) Fraction of the gas stream that is liquefied.

The vapour mass fraction x_D at point D in Figs. 1 and 2 is computed from the lever rule in the two-phase system, $h_D = h_{DI} + x_D(h_{Dv} - h_{DI})$, the energy balance from C to D, $h_C = h_D$, and the energy balance at the liquefier-coil, $x_D(h_A - h_{Dv}) = h_B - h_C$, or simply from the liquefier energy balance $h_B = (1 - x_D)h_{DI} + x_D h_A$:

$$x_D = \frac{h_B - h_{DI}}{h_A - h_{DI}}$$

where it is clear that the perfect-gas model is not applicable ($h_B = h_A$ would yield $x_D = 1$, i.e. no liquid). With values from the nitrogen data in Fig. 2, $h_A = 300$ kJ/kg, $h_B = 270$ kJ/kg, $h_{DI} = -120$ kJ/kg and $h_{Dv} = 78$ kJ/kg, what yields $x_D = 0.93$, whereas from more accurate tabulations one gets $h_A = 446.3$ kJ/kg, $h_B = 414.5$ kJ/kg, $h_{DI} = 25.67$ kJ/kg and $h_{Dv} = 223.2$ kJ/kg, what yields $x_D = 0.924$. Notice the difference in reference value from both sets of data.

- d) Demanded power and exergy efficiency of the plant.

With the perfect-gas model, each compressor is equally loaded and the global power is five times that of the first compressor. The gas flow-rate through the compressors is:

$$\dot{m}_{gas} = \frac{\dot{m}_{liq}}{1 - x_D} = \frac{0.028}{1 - 0.924} = 0.37 \text{ kg/s}$$

and the exit temperature:

$$T_1 = T_0 \left[1 + \frac{\left(\frac{p_1}{p_0} \right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_C} \right] = 288 \left[1 + \frac{\left(\frac{270}{100} \right)^{\frac{1.4-1}{1.4}} - 1}{0.86} \right] = 399 \text{ K}$$

what yields, with the perfect substance model:

$$\dot{W}_{tot} = 5 \dot{m}_{gas} c_p (T_1 - T_0) = 5 \cdot 0.37 \cdot 1 (399 - 288) = 205 \text{ kW}$$

and an exergy efficiency of:

$$\eta_x \equiv \frac{\dot{W}_{min}}{\dot{W}_{comp}} = \frac{19.5}{205} = 0.095$$

Comments:

In practice, liquid nitrogen is not obtained from pure nitrogen gas but from air, by fractional distillation of liquid air, which is obtained by a modification of this simple Linde gas-liquefaction process.

[Back](#)