CHILL WATER REFRIGERATOR

Statement

A simple refrigerating machine using R134a is used to cool a stream of 5 L/min of water from 15 ºC to 5 ºC. Evaluate:

a) Heat to take off and minimum work required.

b) Working pressures, letting 5 ºC of temperature-jump in the heat exchangers.

c) Power needed with a compressor of 70% isentropic efficiency.

d) Energy and exergy efficiencies.

🇪🇸Se quiere enfriar un gasto de 5 L/min de agua desde 15 ºC hasta 5 ºC con una máquina refrigerante simple de R134a. Se pide:

a) Calor a evacuar y trabajo mínimo necesario.

b) Presiones de funcionamiento para dejar un salto mínimo de 5 ºC en los cambiadores.

c) Potencia consumida con un compresor de rendimiento 0,7.

d) Rendimientos energético y exergético de la instalación..

Solution

a) Heat to take off and minimum work required.

To cool a stream of 5 L/min of water from 15 ºC to 5 ºC, the heat to take off is:

$$\dot{Q}_w = \dot{m}_w c_w \left(T_{w1} - T_{w2}\right) = \left(5/60\right)4200\left(15-5\right) = 3.5\text{ kW}$$

where the density of water has been taken as 1 kg/L, and the subscript *w* is used to label that fluid, to later leave unsubscripted the variables related to the refrigerant fluid, R134a.

The minimum work required to pump out that amount of heat (to the environment) is the flow of exergy:

$$\dot{W}_{u,\min} = \dot{m}_w \Delta\psi = \dot{m}_w\left(\Delta h_t - T_0\Delta s\right) = \dot{m}_w c_w\left(T_{w2} - T_{w1} - T_0\ln\frac{T_{w2}}{T_{w1}}\right) =$$
$$= \left(5/60\right)4200\left(278-288-288\ln\frac{278}{288}\right) = 62\text{ W}$$

b) Working pressures, letting 5 ºC of temperature-jump in the heat exchangers.

This refers to the refrigerant: R134a (tetrafluoroethane, CF_3CH_2F). The simple vapour-compression refrigeration cycle assumes that the compressor aspirates saturated vapours of the working fluid, and the state before the throttling valve is saturated liquid. Letting 5 ºC of temperature-jump in the heat exchangers means that the evaporator must work at 0 ºC (5-5) and the condenser at 20 ºC (15+5). Using Antoine correlation for the vapour pressure curve, with fitting values from tables:

$$p_1 = p_v\left(T_1\right) = p_u \exp\left(A - \frac{B}{C + T_1/T_u}\right) = \left(1\ \text{kPa}\right)\exp\left(14.41 - \frac{2094}{-33.06 + 273/1}\right) = 294\ \text{kPa}$$

and similarly $p_2=p_3=p_v(293\ \text{K})=575$ kPa. A sketch of the main components of the machine, and of the processes followed by the refrigerant fluid, follows.

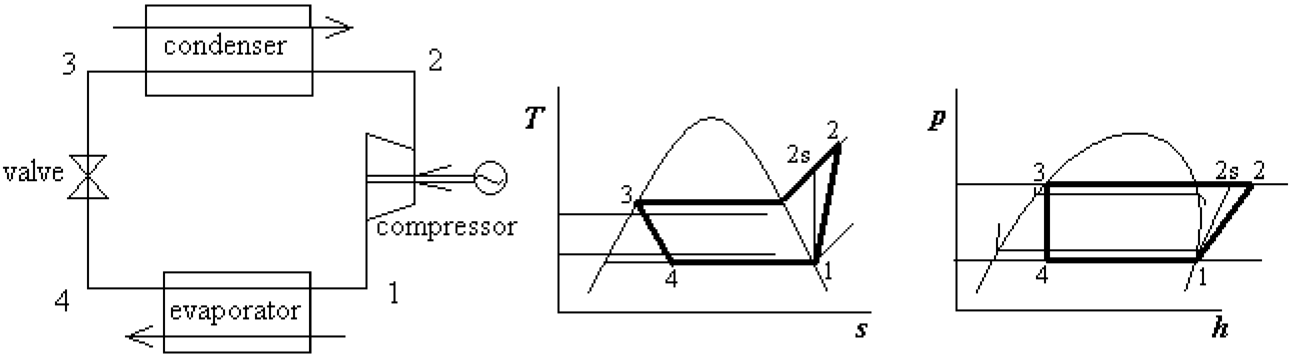

Fig. 1. Sketch of components and flows, *p-h* and *T-s* diagram for the processes.

e) Power needed with a compressor of 70% isentropic efficiency.

We solve the problem using the perfect substance model because of its simplicity for routine computations (perfect gas, perfect liquid, vapour-pressure curve and vaporisation enthalpy), but we check afterwards with most precise tabulated data. A graphical solution using a *p-h* diagram of the working fluid is even best for a manual computation, as later shown.

If we take as a reference for enthalpy and entropy the liquid state at the triple point (roughly the saturated liquid at the normal freezing point), $T_{ref}=177$ K, the enthalpy values at the key points are: $h_1=c(T_1-T_{ref})+h_{lv}(T_1)=c(T_1-T_{ref})+h_{lv}(T_b)-(c-c_p)(T_1-T_b)=$ 1300·(273−177)+ 215000−(1300−840)·(273-247)=328 kJ/kg, $h_3=c(T_3-T_{ref})=1300\cdot(293-177)=151$ kJ/kg, $h_4=h_3$, and h_2 is obtained from the compressor isentropic efficiency:

$$\eta_C \equiv \frac{\dot{W}_s}{\dot{W}} = \frac{\dot{m}\left(h_{2s}-h_1\right)}{\dot{m}\left(h_2-h_1\right)} \overset{PGM}{=} \frac{T_{2s}-T_1}{T_2-T_1} = \frac{\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1}{\dfrac{T_2}{T_1}-1} \rightarrow$$

with $\gamma=c_p/c_v=c_p/(c_p-R)=840/(840-8.3/0.102)=1.107$. Solving for T_2:

$$T_2 = T_1\left[1+\frac{\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}-1}{\eta_C}\right] = 273\left[1+\frac{\left(\dfrac{575}{294}\right)^{\frac{1.107-1}{1.107}}-1}{0.7}\right] = 299\ \text{K}$$

what yields, with the perfect substance model, $h_2=h_1+c_p(T_2-T_1)=328+0.84\cdot(299-273)=350$ kJ/kg. Power consumption by the compressor is $\dot{W}=\dot{m}_R\left(h_2-h_1\right)$, but we need first to compute the mass flow-rate of the refrigerant. From the energy balance of the evaporator:

$$\dot{m}_R\left(h_1-h_4\right)=\dot{Q}_w \longrightarrow \dot{m}_R=\frac{\dot{Q}_w}{h_1-h_4}=\frac{3.5}{328-151}=0.020\text{ kg/s}$$

and thence $\dot{W}=0.020\left(350-328\right)=0.44\text{ kW}$.

This result may be compared with more exact values (e.g. REFPRO from NIST, USA), from which one gets p_1=291 kPa, p_3=569 kPa, h_1=233 kJ/kg, h_3=61.4 kJ/kg, s_1=858 J/(kg·K), T_{2s}=296 K, h_{2s}=246 kJ/kg, $h_2=h_1+(h_{2s}-h_1)/\eta=233+(246-233)/0.7=252$ kJ/kg, and from them $\dot{m}_R=\dot{Q}_w/\left(h_1-h_4\right)=0.020\text{ kg/s}$ and $\dot{W}=\dot{m}_R\left(h_2-h_1\right)=0.020\left(252-233\right)=0.38\text{ kW}$. The error may seem large (16%), but it is in the conservative side, i.e. it asks for a more powerful compressor, what is handy if one realises that pressure losses in the piping, heat transfer on them, and other irreversibilities have been not accounted for.

Notice the difference in the reference state implied in the two cases: we selected T_{ref}=177 K (the normal freezing point), whereas T_{ref}=247 K (the normal boiling point) was used in REFPRO. The variety is great; the ASHRAE society usually chooses T_{ref}=–40 ºC. See also a graphical solution in Fig. 2.

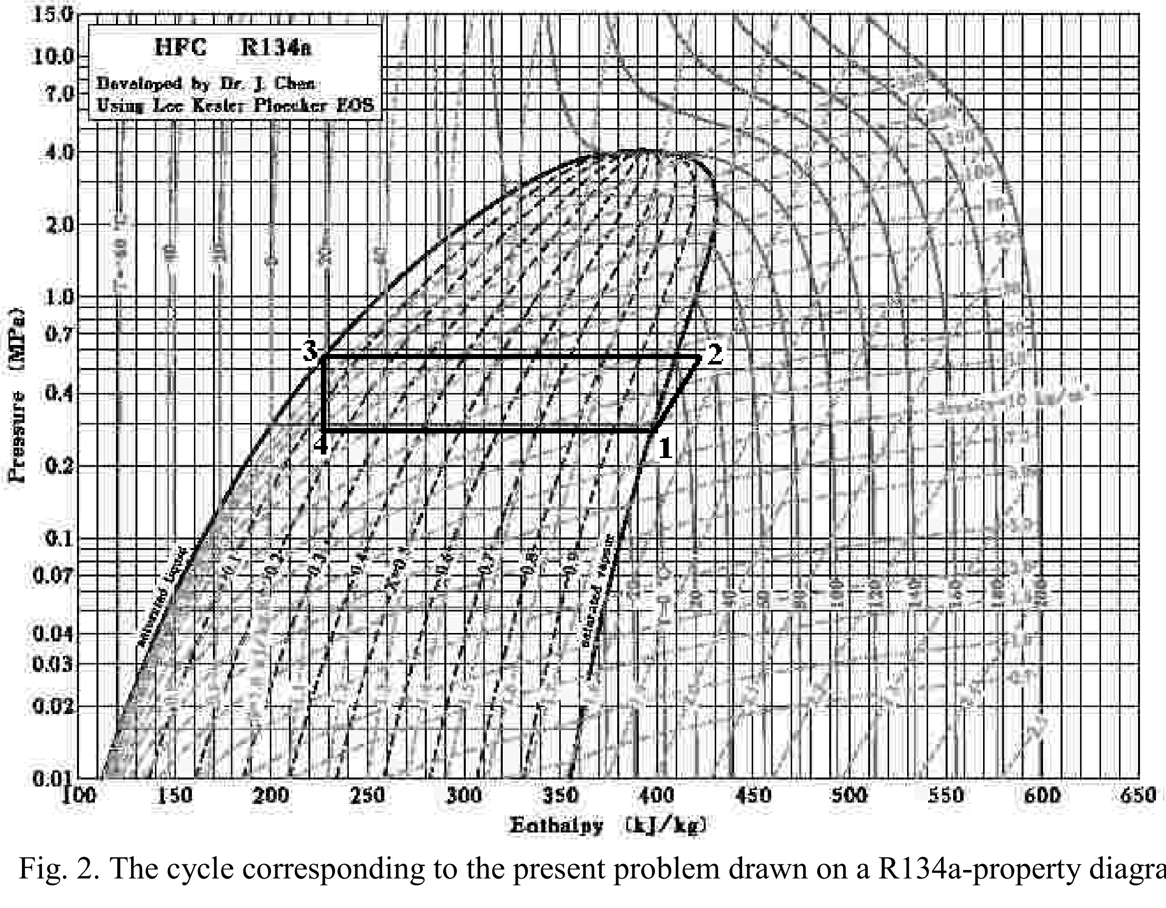

Fig. 2. The cycle corresponding to the present problem drawn on a R134a-property diagram.

d) Energy and exergy efficiencies.

Energy efficiency measures the heat pumped off the load, relative to the work consumed:

$$\eta_e = \frac{\dot{Q}_{load}}{\dot{W}_{comp}} = \frac{\dot{m}_R\left(h_1 - h_4\right)}{\dot{m}_R\left(h_2 - h_1\right)} = \frac{328-151}{350-328} = 8.0$$

or η_e=9.0 using REFPRO data.

Exergy efficiency measures the work consumed relative to the minimum required work (i.e. to the thermodynamic limit:

$$\eta_x \equiv \frac{\dot{W}_{u,\min}}{\dot{W}_{comp}} = \frac{62\ \text{W}}{440\ \text{W}} = 0.14$$

or η_x=0.16 using REFPRO data

Comments

Notice that the energy efficiency does not give a good idea of the engineering achievement; in our case it might seem impressive to have a 800% (or 900%) energy efficiency, but the real measure of thermodynamic perfection is exergy efficiency, 14% (or 16%) in our case; not so brilliant as achievement.

Back