DOMESTIC REFRIGERATOR

Statement

An old domestic refrigerator works with R12 between 150 kPa and 950 kPa, with a mass flow-rate of 12 kg/h and a compressor isentropic efficiency of 70%. Evaluate:

a)   Sketch of components and flows, *p-h* and *T-s* diagram for the processes, and mass fraction at the valve exit.

b)   Refrigeration capacity and minimum working temperature.

c)   Energy and exergy efficiencies.

Un viejo frigorífico doméstico de R12 funciona con presiones de 150 kPa y 950 kPa, con un gasto másico circulante de 12 kg/h y un rendimiento isentrópico de compresión del 70%. Se pide:

a)   Esquema de componentes, diagramas *p-h* y *T-s* de los procesos, y fracción másica de vapor a la salida de la válvula.

b)   Capacidad de refrigeración y temperatura mínima práctica de trabajo.

c)   Rendimientos energético y exergético de la instalación.

Solution

a)   Sketch of components and flows, *p-h* and *T-s* diagram for the processes, and mass fraction at the valve exit.

There are four main components in a common refrigeration machine (vapour compression type): compressor, condenser, valve and evaporator. Assuming a typical inverse-Rankine cycle, i.e., with saturated liquid entering the valve and saturated vapour entering the compressor, the sketch is:
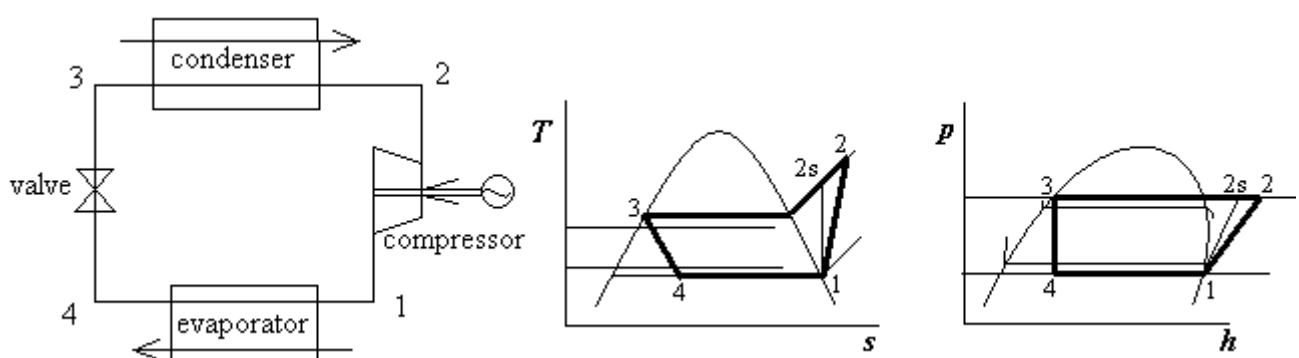

Fig. 1. Sketch of components and flows, *p-h* and *T-s* diagram for the processes.

To find the vapour mass-fraction at point 4, one needs the thermodynamic properties of the R12 fluid (dichlorodifluoromethane, CCl_2F_2), what may come as a graphic chart (R12 charts came in all Thermodynamics books of the 20th century), as tabulated data, as key data to build a simple model like the perfect substance model, as a data-fitting algorithm, or as a closed computer routine. We here use the perfect substance model, where the liquid is assumed incompressible and with constant thermal capacity, the vapour is assumed an ideal gas with constant thermal

capacity, and a vaporisation enthalpy is known, i.e. we rely on just the following data set: $\rho = pM/(R_uT)$ with $M=0.121$ kg/mol and $c_p=573$ J/(kg·K) for the vapour, $\rho=1330$ kg/m³ and $c=966$ J/(kg·K) for the liquid, and $h_{lv}=165$ kJ/kg for the vaporisation enthalpy at the normal boiling point, $T_b=243$ K.

Vapour mass fraction at point 4 is computed from the energy balance from 3 to 4, $h_3=h_4$ (i.e. an isenthalpic process), and the lever rule from $h_4= h_{4l}+x_4(h_{4v}-h_{4l})$:

$$x_4 = \frac{h_4 - h_{4l}}{h_{4v} - h_{4l}}$$

We have to choose a reference for energies and entropies, although the measurable results ($q+w=\Delta h$) do not depend on our choice. Several standards are in use:

- IIR (International Institute of Refrigeration) reference sets $h=200$ kJ/kg and $s=1$ kJ/(kg·K) for the saturated liquid at $T=0$ ℃.
- ASHRAE (American Society of Heating, Refrigerating and Air Conditioning Engineers) reference sets $h=0$ and $s=0$ for the saturated liquid at $T=-40$ ℃ (=-40 ℉).
- Others set $h=0$ and $s=0$ for saturated liquid at its normal boiling point, but a better choice (to avoid negative values) is setting $h=0$ and $s=0$ for the saturated liquid state at the triple point.

To show the arbitrariness of the reference choice, we here set $h=0$ and $s=0$ for the saturated liquid at $T_{ref}=0$ ℃. With the perfect-substance model, $h_{4l}=c(T_4-T_{ref})$ and $h_{4v}=h_{4l}+h_{lv}(T_4)=h_{4l}+h_{lv}(T_b)-(c-c_p)(T_4-T_b)$, where the vaporisation enthalpy at any temperature is set as a linear-slope approximation from the boiling point. But we have to find T_4 from the pressure data. Using Antoine correlation for the vapour pressure curve,

$$p_v(T_4) = p_u \exp\left(A - \frac{B}{C+T_4/T_u} \right) \rightarrow T_4 = T_u\left(\frac{B}{A - \ln\dfrac{p_v(T_4)}{p_u}} - C \right) =$$

$$= (1\ \text{K})\left(\frac{1971}{13.79 - \ln\dfrac{150}{1}} - (-28.30) \right) = 253\ \text{K}$$

and, similarly, for $p_v(T_3)=950$ kPa one gets $T_3=313$ K. Substitution yields: $h_4=h_3=c(T_3-T_{ref})=966(313-273)=38.6$ kJ/kg, $h_{4l}=c(T_4-T_{ref})=966(253-273)=-20.2$ kJ/kg, $h_{lv}(T_4)=h_{lv}(T_b)-(c-c_p)(T_4-T_b)=165-(966-573)(253-243)=161$ kJ/kg, $h_{4v}=h_{4l}+h_{lv}(T_4)=-20.2+161$ $=141$ kJ/kg, and finally $x_4=(h_3-h_{4l})/(h_{4v}-h_{4l})=(38.6+20.2)/(141+20.2)=0.36$.

It can be checked that the result is independent on the choice of reference state.

b) Refrigeration capacity and minimum working temperature.

Refrigeration capacity is $\dot{Q} = \dot{m}(h_1 - h_4) = (12/3600)(141-38.6)=340$ W.

With the evaporator at 253 K (−20 °C), it is clear that the coldest temperature achievable by heat transfer from the load has that limit, but in practice, a temperature jump must be allowed to have a finite-size heat exchanger; if 5 °C are allowed, the load might be cooled down to −15 °C.

c) Energy and exergy efficiencies.

The energy efficiency is defined as the quotient of energy evacuated from the cold side (the benefit of a refrigerator) to energy spent in the process (the cost of running the compressor), i.e.:

$$\eta_e \equiv \frac{\dot{Q}_{14}}{\dot{W}_{12}} = \frac{\dot{m}(h_1 - h_4)}{\dot{m}(h_2 - h_1)}$$

where h_2 is obtained from the compressor isentropic efficiency:

$$\eta_C \equiv \frac{\dot{W}_s}{\dot{W}} = \frac{\dot{m}(h_{2s} - h_1)}{\dot{m}(h_2 - h_1)} \overset{PGM}{=} \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{T_2}{T_1} - 1} \rightarrow$$

with $\gamma = c_p/c_v = c_p/(c_p - R) = 1.14$. Solving for T_2:

$$T_2 = T_1\left[1 + \frac{\left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_C}\right] = 253\left[1 + \frac{\left(\dfrac{950}{150}\right)^{\frac{1.14-1}{1.14}} - 1}{0.7}\right] = 342 \text{ K}$$

what yields, with the perfect substance model, $h_2 = h_1 + c_p(T_2 - T_1) = 141 + 0.573(342 - 253) = 192$ kJ/kg.

Power consumption by the compressor is then:

$$\dot{W} = \dot{m}(h_2 - h_1) = (12/3600)(192 - 141) = 190 \text{ W}$$

and energy efficiency (also known as COP):

$$\eta_e \equiv \frac{\dot{Q}_{14}}{\dot{W}_{12}} = \frac{340}{190} = 1.8$$

Exergy efficiency measures the work consumed relative to the minimum required work (i.e. to the thermodynamic limit, that corresponds to an ideal inverse-Carnot cycle between the ambient temperature and the load temperature; if we assume those values to be 288 K and

(253+5) K, Carnot energy efficiency and exergy efficiency would be:

$$\eta_{e_{\text{Carnot}}} = \frac{Q_{load}}{W_{\min}} = \frac{T_{load}}{T_{amb} - T_{load}} = \frac{258}{288 - 258} = 8.6$$

$$\eta_x \equiv \frac{\eta_e}{\eta_{e_{\text{Carnot}}}} = \frac{1.8}{8.6} = 0.21$$

Notice that the choice of the ambient temperature has an important effect on those values; in practice, normalisation institutions establish temperature standards to ensure a rational comparison of published characteristics (but there are several standardising bodies).

Comments

The perfect substance model (PSM) here used does not give accurate results when the fluid is working at very high pressures (near its critical region), as may be seen in a *p-h* diagram by the bending of the isotherms (of the isenthalpics in the *T-s* diagram). To ascertain the accuracy of the model here, we solve now the problem using the most precise data at hand (a large tabulation of R12 data).

From saturated values at 150 kPa, T_1=253.0 K, h_l=8.623 kJ/kg (the reference is surely different), h_v=170.7 kJ/kg and , s_v=675.2 J/(kg·K). At 950 kPa, T_1=312.8 K, h_l=65.95 kJ/kg.
Interpolation at 950 kPa for an isentropic process from 675.2 J/(kg·K) yields h_{2s}=203.6 kJ/kg, and from the isentropic coefficient definition $h_2=h_1+(h_{2s}-h_1)/h_C$=170.7+(203.6−170.7)/0.7=217.7 kJ/kg. Thus:

$$x_4 = \frac{h_4 - h_{4l}}{h_{4v} - h_{4l}} = \frac{65.95 - 8.623}{170.7 - 8.623} = 0.354.$$

$$\dot{Q} = \dot{m}(h_1 - h_4) = (12/3600)(170.7 - 65.95) = 349 \text{ W}.$$

$$\dot{W} = \dot{m}(h_2 - h_1) = (12/3600)(217.7 - 170.7) = 157 \text{ W}$$

$$\eta_e = \frac{h_1 - h_4}{h_2 - h_1} = \frac{170.7 - 65.95}{217.7 - 170.7} = 2.23$$

$$\eta_x \equiv \frac{\eta_e}{\eta_{e_{\text{Carnot}}}} = \frac{2.23}{8.6} = 0.26$$

In conclusion, an acceptable approximation for a first analysis (only in the required power there is a 20% bias, but on the safe side, i.e. that could compensate other losses not accounted for in this preliminary analysis, as pressure drops in heat exchangers and piping).