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PROPELLERS

A [propeller](#) (more explicitly named as blade propeller or screw propeller) is a propulsion device that creates relative motion by forcing the environmental fluid axially backwards, by means of azimuthally-rotating blades mounted in a shaft that is driven by an engine (reciprocating, rotational, turbine, electrical...). A pressure difference is produced between the forward and rear surfaces of the [aerofoil](#)-shaped blades, producing thrust that is transmitted from the blades to the shaft, and finally to the vehicle through the engine supports. A similar device, but mounted on a fix support instead of a vehicle, is named '[mechanical fan](#)'.

Propellers are being used since the late 19th century in ships, submarines, airships, aircraft, helicopters, hovercraft, windmills, turbines, pumps, stirrers.... Although the origin of the screw propeller starts with Archimedes' screw in the 3rd century BCE, James Watt is generally credited with applying the first screw propeller around 1770 (to a steam engine on a ship). Alberto Santos Dumont in the late 1890s used propellers for his airships, but they were untwisted along the length of the blade (very inefficient). The Wright brothers developed the [twisted blade](#) for their Flyer-I in 1903 (contrary to later aircraft, Flyer-I had the propellers [behind](#) the wings). A few different types of propellers are shown in Fig. 1.

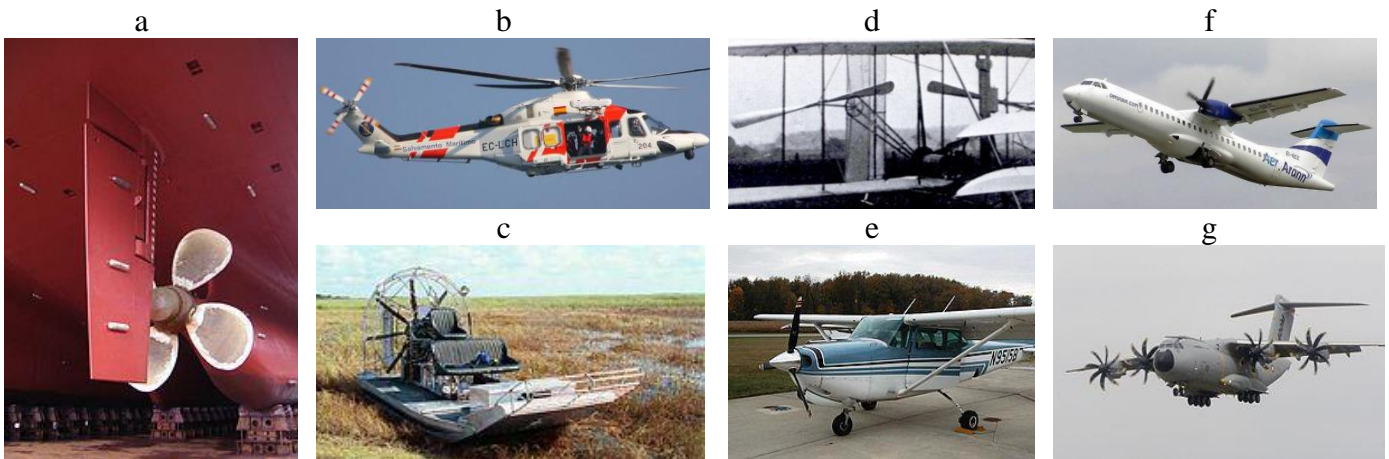


Fig. 1. Different types of propellers: a) [Ship](#) propeller and rudder. b) Two propellers in a [SAR helicopter](#) (main and tail rotors). c) Eight-blade propeller in an Everglades [air-boat](#). d) Original two-blade [Flyer-I](#) propellers. e) Three-bladed propeller of a [Cessna 172](#). f) Twin six-blade propeller in [ATR-72](#) regional [turboprop](#). g) Four counter-rotating eight-bladed scimitar-type propellers in [A400M](#).

Propellers are not used for terrestrial propulsion (road or rail) because solid friction with wheels is far more efficient than any fluid-based means (either with propellers or with jet engines; with a wheel, the whole Earth is pushed back, instead of a small amount of fluid; see [Propulsion fundamentals](#)).

PARAMETERS AND PERFORMANCES

The geometry of a propeller is usually defined by a set of simple parameters: propeller size is quantified by tip diameter, D ; advance pitch, p , is the theoretical axial displacement in one turn as if a reference blade-section was screwed on a solid (changeable in variable pitch propellers; in a fix-pitch propeller with twisted blades, the pitch varies radially along the blades); the number of blades, N_B (all blades have the same blade-geometry); disc area ratio DAR is defined as blades area divided by $\pi D^2/4$; hub area; direction of rotation (right/left handed; a propeller that turns clockwise to produce forward thrust when viewed from aft, is called right-handed); rake angle is the angle of the blade leading-edge with the perpendicular to the hub, viewed side on; positive if the blade slopes downwards); skew angle (for non-radially symmetric blades)... When more detailed blade data is needed, the chord and pitch-angle radial distributions are given, $c(r)$ and $\theta(r)$, or the full 3D surface specification.

For a built propeller of fix geometry, there are two basic operational parameters: spinning rate, n , and advance speed, v_0 (relative axial speed to the undisturbed fluid); the spinning or rotation rate may be stated in Hz (i.e. revolutions per second, rps, though [rpm](#) is more common), or as angular speed, ω ($\omega=2\pi n$, for n in rps). With these two basic operational parameters, a non-dimensional parameter named [advance ratio](#), J , is defined by $J \equiv v_0/(nD)$, i.e. the distance advanced by the propeller in one revolution made dimensionless with propeller diameter.

Many propellers, however, have another degree of freedom, the theoretical advance pitch (or [blade pitch](#)) either measured as the penetration distance per revolution, p (as for a corkscrew), or as the reference blade-chord angle, θ , relative to the propeller plane; they are both related by $p=(3/4)\pi D \arctan \theta$ (the $(3/4)D$ is the reference radial distance along the blade axis to define blade chord and blade pitch, arbitrarily chosen at 3/4 of the blade length; recall that the chord of a blade is the distance from the leading edge to the trailing edge). Variable pitch propellers are built with independent blades, all of them rotatable along their longitudinal axis with a mechanism at the hub.

Aerodynamic effects cause real propellers (working within a fluid, not within a solid) to advance at a speed $v_0 < pn$ (i.e. advance speed is lower than pitch times rps); the difference $(pn - v_0)$ is named slip speed, and $S \equiv (pn - v_0)/(pn)$ is termed slip ratio. Pitch should be increased as advancing speed increases, to keep propeller efficiency, $\eta_p \equiv Fv_0/\dot{W}_{\text{shaft}}$, to a maximum, usually assumed as $\eta_{p,\text{max}}=0.8$, see Fig. 2 (propeller efficiency in ships is smaller, $\eta_p=0.5..0.7$, the smaller for large tankers, which have advance ratios $J=0.2..0.4$).

Besides propeller advance ratio J , and propeller efficiency η_p , thrust and power coefficients for a propeller are defined as follows, and typical values presented in Fig. 2.

Advance ratio: $J \equiv \frac{v_0}{nD}$

Efficiency: $\eta_p \equiv \frac{Fv_0}{\dot{W}_{shaft}} = \frac{c_F}{c_P} J$

Thrust coefficient: $c_F \equiv \frac{F}{\rho n^2 D^4}$

Power coefficient: $c_P \equiv \frac{\dot{W}_{shaft}}{\rho n^3 D^5}$

(1)

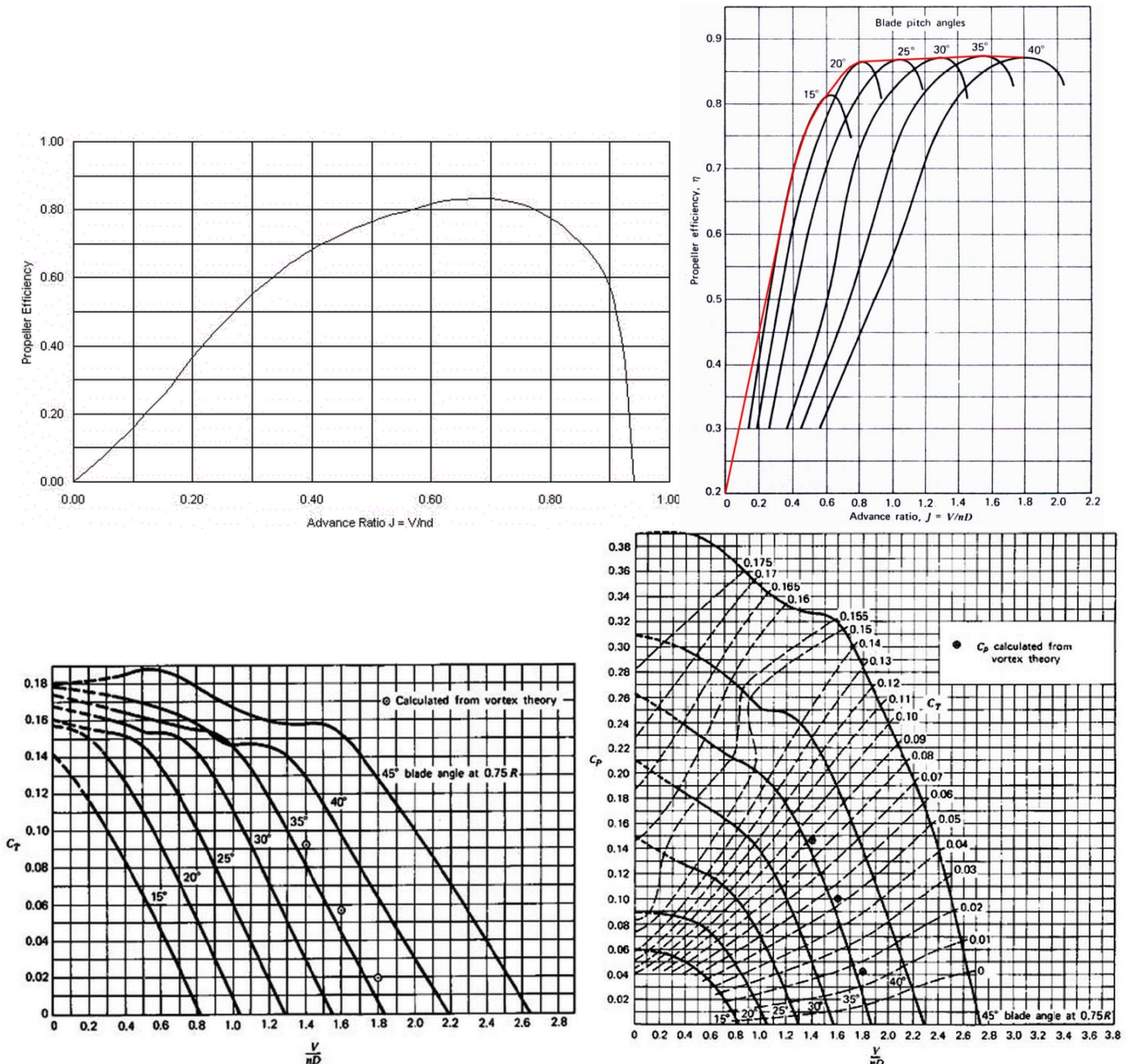


Fig. 2. Typical propeller characteristics: a) efficiency (η_p) vs. advance ratio (J) for a fix-pitch propeller ([McCauley 7557 propeller on some Cessna 172s](#)). b) Variation with pitch-angle of a variable-pitch propeller (McCormick-1979). c) Variation with advance ratio J and pitch-angle θ of the coefficient of thrust c_F (McCormick-1979). d) Variation with advance ratio J and pitch-angle θ of the coefficient of power c_P ([McCormick-1979](#)).

Exercise 1. An airboat (or fan-boat) is a flat-bottomed vessel propelled by an aircraft-type propeller (driven by a reciprocating engine) to sail in shallow or marshy waters (Fig. 1c). For a certain

propeller of 2 m in diameter spinning at 1200 rpm, thrust and power coefficients can be approximated in terms of advance ratio J as $c_F=0.05-0.06J-0.1J^2$, and $c_P=0.015-0.03J^2$, respectively. If boat drag can be approximated as $D=c_D A \rho_w v_0^2/2$, with $c_D=0.001$, $A=3 \text{ m}^2$, ρ_w water density, and neglecting air drag (assumed included in boat drag), find:

- Propeller efficiency versus advance ratio, and advance speed for maximum efficiency.
- Power demanded by the propeller, and thrust provided, at zero advance speed.
- Maximum advance speed, and corresponding values of η_p , F , and \dot{W}_{shaft} .

Sol.: a) Using equations (1) [we obtained](#) for the propeller efficiency the plot in Fig. E1a; the maximum is $\eta_p=0.56$, at $J=0.29$, corresponding to an advance speed of $v_0=JnD=12 \text{ m/s}$.

b) With $v_0=J=0$, and for standard sea-level conditions with $\rho=1.21 \text{ kg/m}^3$ for air, we get $F=c_F \rho n^2 D^4=390 \text{ N}$, and $\dot{W}_{\text{shaft}}=c_P \rho n^3 D^5=4.7 \text{ kW}$.

c) At start, the propeller yields a thrust of $F=390 \text{ N}$ without any opposing force (no drag), and the airboat accelerates until drag matches thrust (Fig. E1b); i.e. maximum v_0 corresponds to $F=D$ (thrust=drag), what solves to yield $J=0.28$ ($v_0=11 \text{ m/s}$). At this point, $F=190 \text{ N}$, $\dot{W}_{\text{shaft}}=3.9 \text{ kW}$, and $\eta_p=0.56$.

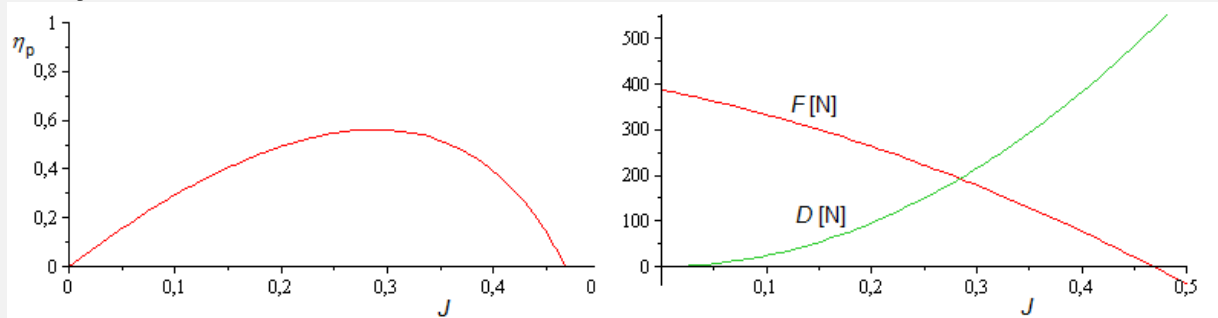


Fig. E1. Propeller efficiency (η_p), propeller thrust (F), and airboat drag (D) vs. advance ratio (J).

The purpose of a propeller is to produce thrust, F , driven by a shaft power $\dot{W}_{\text{shaft}} = Q\omega$, where Q is the torque and ω the angular speed (wind turbines and water turbines are very similar, but their purpose is to produce power from the flow, i.e. to absorb energy from the fluid, instead of giving in energy to the fluid for propulsion). It is found below that power consumed to drive a propeller, which under nominal operation gives a measure of the propulsive power, is directly proportional to pitch, to the third power of revolutions, and to the fifth power of diameter, i.e. $\dot{W}_{\text{shaft}} \propto \theta n^3 D^5$, showing the great importance of propeller diameter D (the larger the better, limited by available space or by the speed of the blade tip), and of spinning rate, n . But diameter is fixed for a given propeller, and engine speed can only vary a little for good engine-performances, so that the best way to control to control advance speed, thrust, and power in a propeller is the pitch angle.

Pitch angle for best efficiency is usually in the range $\theta=20^\circ..25^\circ$ (recall, measured at 3/4 of the blade length). High efficiency in air-propellers is obtained at high rotation rates but with tip speed below the speed of sound (to avoid transonic losses), whereas in water-propellers tip speed is limited by cavitation (to avoid blade surface damage). Blade pitch acts much like the [gearing](#) in car driving: low pitch yields best low-speed acceleration (and climb rate in an aircraft) while high pitch optimizes high speed performance and economy (like a high gear). For a given shaft power, if the propeller has in-flight adjustable pitch, a low pitch is set for maximum take-off thrust (the blades give large lift with low drag),

and a higher pitch for optimal cruising efficiency (large F_{v_0}); besides, in case of engine stop in flight, the blades can be rotated to become parallel to the airflow ([feathering](#)) to reduce drag and to avoid the engine-shaft being forced to turn). Pitch is automatically adjusted to keep the engine and the propeller operating at a constant rpm (e.g. to climb, pitch is decreased; in a dive, pitch is increased). Most controllable-pitch and constant-speed propellers also are capable of being reversed. This is done by rotating the blades to a negative or reverse pitch. Reversible propellers push air forward, reducing the required landing distance as well as reducing wear on tires and brakes.

The number of blades, N_B , in a propeller has little effect on performances (for a given shaft power). A one-blade propeller (with a counterweight, of course) is used in some model airplanes. Two slender blades were most used in early developments like Wright's brother Flyer-I of 1903 (Fig. 1d), and are still used on simple applications, or when drag is to be minimised. Three blades seems to be the best compromise between aerodynamic and structural concerns in low-speed applications (boats, small aircraft like in Fig. 1e, windmills), but for higher spinning rates, four to eight blades are better (Fig. 1f and g), since they accelerate quicker and have less vibrations. In [propfans](#), where only the farther blade-sections are working, eight or more blades are better. In general, more powerful engines require more propeller blades. However, a multi-bladed propeller has more induced drag, caused by tip vortices (air spilling over the blade tips, just like wingtip vortices on a wing), because there are more tips. Hence, overall efficiency is lower, in much the same way that a biplane (even one without struts and bracing wires) is less efficient than a monoplane with the same wing area. Solidity is the frontal disc-area fraction occupied by the blades, and can be increased not only by more blades but by larger blade-chords, although the latter decreases blade slenderness and aerodynamic efficiency. Sweeping the blade tip back (scimitar blade, Fig. 1g) reduces the noise generated and may also improve the efficiency. Both on air-propellers and water-propellers, the tip sections should not be too loaded, i.e. aerofoil lift coefficient c_L should be limited to $c_L < 0.5$, to avoid transonic flow in air, or [cavitating](#) flow in water.

When using a single propeller, the craft may be already designed to compensate some of the angular momentum effects (torque reaction, gyroscopic precession, twisted down-stream, and lateral-wind counteraction in opposite blades), at least on cruise (except in helicopters, where a tail rotor is commonly used). Traditionally, a single aircraft propeller rotates anticlockwise when looking in front.

In crafts with paired engines, symmetrical engines used [counter-rotating](#) propellers (in parallel). Not to be confused with [contra-rotating](#) propellers (i.e. two propellers in series in the same axis), which have increased efficiency by capturing the energy lost in the tangential velocities imparted to the fluid by the forward propeller (i.e. the propeller swirl).

Several theories have been developed to model propellers performances; from the simplest to more complex, we have the disk-actuator theory, blade-element theory, wake methods, to 3D time-averaged Navier-Stokes equations.

Disk actuator theory

The [disc actuator theory](#) is an ideal momentum model (no friction, incompressible, and irrotational) developed by W.J.M. Rankine (1865) and R.E. Froude (1889). The propeller is assumed to be a uniform permeable disc of the same swept area, with virtual devices introducing a pressure jump across (i.e. uniformly distributed singularities) representing the total thrust; it might be thought of as a limit for infinite number of very thin blades. The one-dimensional axial balance equations are (Fig. 3):

- Mass balance: $\dot{m} = \rho v_0 A_0 = \rho v_1 A = \rho v_2 A = \rho v_3 A_3$.
- Momentum balance: $F = (p_2 - p_1)A = \dot{m}(v_3 - v_0) = \rho v_3 A_3 (v_3 - v_0)$, since, in the ground reference frame, there is only a momentum outflow at 3.
- Energy balance (Bernoulli): from 0 to 1 is $p_1 = p_0 - \frac{1}{2}\rho(v_1^2 - v_0^2)$, from 1 to 2 is $\dot{W}_{\text{shaft}} = Fv_1$, and from 2 to 3 is $p_2 = p_3 + \frac{1}{2}\rho(v_3^2 - v_2^2)$ with $p_3 = p_0$.

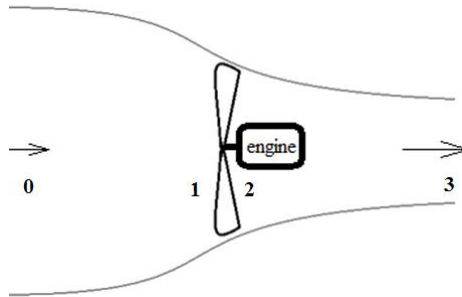


Fig. 3. Disc actuator model.

Solving in terms of the so called induced speed, $v_i \equiv v_1 - v_0$ one gets: $v_3 = v_0 + 2v_i$, $F = 2\rho A(v_0 + v_i)v_i$, $\dot{W}_{\text{shaft}} = Fv_1 = Fv_0 + Fv_i$, $\eta_p \equiv Fv_0 / \dot{W}_{\text{shaft}} = 1/(1 + v_i/v_0)$, with:

$$v_i = \frac{1}{2} \left[\sqrt{v_0^2 + \frac{2F}{\rho A}} - v_0 \right] \xrightarrow{v_0 \rightarrow 0} \sqrt{\frac{F}{2\rho A}} \longrightarrow F|_{v_0 \rightarrow 0} = (2\rho A \dot{W}_{\text{shaft}}^2)^{\frac{1}{3}} \quad (2)$$

Parameter F/A , named disc loading, is the average pressure change across an actuator disk. In a hovering helicopter is the ratio of its weight to the total main rotor disc area. Notice that half of the flow acceleration occurs by suction in front, and half by blowing to the rear.

For $v_0 \neq 0$ one may introduce the parameters $a \equiv v_i/v_0$ and write the above results as:

$$a \equiv \frac{v_i}{v_0} = \frac{1}{2} \left[\sqrt{1 + \frac{F/A}{\frac{1}{2}\rho v_0^2}} - 1 \right], \quad \eta_p \equiv \frac{Fv_0}{\dot{W}_{\text{shaft}}} = \frac{2}{1 + \sqrt{1 + \frac{F/A}{\frac{1}{2}\rho v_0^2}}} = \frac{2}{1 + \sqrt{1 + \frac{2\eta_p \dot{W}_{\text{shaft}}}{\rho v_0^3 A}}} \quad (3)$$

showing that maximum propulsive efficiency is obtained with disc loading (F/A) much smaller than incoming dynamic pressure (ρv_0^2).

If, besides this linear momentum balance, an angular momentum balance is established to account for the actual torque Q (at the disc) forced by the shaft engine, then an additional induced rotational speed appears. This azimuthal component, although smaller than the axial induced speed (as $F \gg Q/R$, or $L \gg D$

for aerofoils), is no longer uniform but radially increasing, so that the analysis is made on non-interacting annular stream tubes (the wake is rotating, but still irrotational), with thrust and torque being:

$$dF = d\dot{m}(v_{3z} - v_0) = \rho dA v_1 (v_{3z} - v_0) = \rho(2\pi r dr) [v_0(1+a)](2av_0) = 4\pi r \rho v_0^2 a(1+a) dr$$

$$dQ = d\dot{m}r(v_\theta - 0) = \rho dA v_1 r (v_\theta - 0) = \rho(2\pi r dr) [v_0(1+a)] r(2b\omega r) = 4\pi r^3 \rho v_0 b \omega (1+a) dr$$

(4)

Exercise 2. Estimate the thrust and specific fuel consumption of the Wright's Flyer-I (1903), Fig. 1d, from the following data: power $\dot{W}_{\text{shaft}} = 9$ kW, engine efficiency $\eta_t = 20$ %, and twin propellers of $D = 2.4$ m in diameter.

Sol.: The actuator disc theory (2) gives for each propeller of $9000/2 = 4500$ W, a thrust of $F = [2\rho(\pi D^2/4) \dot{W}_{\text{shaft}}^2]^{1/3} = [2 \cdot 1.2 \cdot (\pi 2.4^2/4) \cdot 4500^2]^{1/3} = 600$ N, i.e. 1200 N in total. Although the twisted propellers were much more efficient than flat designs of the epoch, and $\eta_p = 0.65$ is often attributed, we take a value of $\eta_p = 0.5$ to account for losses in the chain transmission, and thus take as total real thrust $F = 600$ N.

From the engine efficiency, $\eta_t \equiv \dot{W}_{\text{shaft}} / (\dot{m}_f h_{\text{LHV}}) = 0.20$, with $h_{\text{LHV}} = 44$ MJ/kg typical of gasoline, one gets $\dot{m}_f = 9000 / (0.2 \cdot 44 \cdot 10^6) = 1.0$ g/s. The fuel tank in Flyer-I had a 1.5 L of gasoline in a 6 L tank.

With the above values, brake specific fuel consumption (BSFC) is $c_{\text{sp}} = \dot{m}_f / \dot{W}_{\text{shaft}} = 1/9 = 0.1$ (g/s)/kW = 400 g/kWh, and the thrust specific fuel consumption is $\text{TSFC} = \dot{m}_f / F = 1/0.6 = 1.7$ (g/s)/kN.

Exercise 3. A propeller is being considered for a 4-seat aircraft powered by a 100 kW piston engine (e.g. Cessna 172, Fig. 1e, with a Lycoming O-360 power plant). Three propeller diameters are to be analysed: $D = 1$ m, 2 m and 3 m, and at three advancing speeds: $v_0 = 20$ m/s, 40 m/s, and 60 m/s. For sea-level conditions, and the 9 cases stated, find and comment:

- Propulsion efficiency.
- Thrust.

Sol.:

- Propulsion efficiency.

Using the last of (3) to find η_p (recursively), with $\dot{W}_{\text{shaft}} = 100$ kW, $\rho = 1.21$ kg/m³, and $A = \pi D^2/4$, one gets the following values:

	$v_0 = 20$ m/s	$v_0 = 40$ m/s	$v_0 = 60$ m/s
$D = 1$ m	$\eta_p = 0.44$	$\eta_p = 0.71$	$\eta_p = 0.85$
$D = 2$ m	$\eta_p = 0.62$	$\eta_p = 0.87$	$\eta_p = 0.95$
$D = 3$ m	$\eta_p = 0.72$	$\eta_p = 0.93$	$\eta_p = 0.97$

showing that the propeller efficiency quickly grows with diameter at low speeds, and with advancing speed for small diameters (it is $\eta_p = 0$ at $v_0 = 0$), but the influence at large speeds is moderate.

- Thrust.

Using (3) with the values found for η_p , we get:

	$v_0 = 20$ m/s	$v_0 = 40$ m/s	$v_0 = 60$ m/s
$D = 1$ m	$F = 2.2$ kN	$F = 1.8$ kN	$F = 1.4$ kN

$D=2$ m	$F=3.1$ kN	$F=2.2$ kN	$F=1.6$ kN
$D=3$ m	$F=3.6$ kN	$F=2.3$ kN	$F=1.6$ kN

showing that thrust quickly grows with diameter at low speeds ($F=0$ for $D=0$), and decreases with advancing speed (propellers yield maximum thrust in bench). The influence of diameter at large speeds is almost negligible.

It may be concluded that for low advancing speeds (e.g. airboats, hovercraft, helicopters...) it is very important to use large-diameter propellers.

[Cessna 172](#) is typically equipped with a $D=1.9$ m, 2-bladed, metal propeller.

Blade element theory

The [blade element theory](#), originally designed by William Froude (1878), analyses the 2D aerodynamic forces (i.e. without 3D effects), in an arbitrary blade cross-section at a generic radial distance r , to find its contribution to thrust F and torque Q (Fig. 4). Blades are rotating wings, with similar aerodynamic cross-section (i.e. a thick leading edge and a thin trailing edge, comb...), except near the hub where the cross-section may be almost circular for structural reasons (in the wing/body-joint the distortion is not so great). Blade chord $c(r)$ and pitch angle $\theta(r)$ distributions along the blade length are assumed fixed (i.e. rigid blade). The local angle of attack $\alpha(r)$ is to be found from composition of velocities, the c_L and c_D coefficients assumed known from [airfoil](#) data, and the corresponding dL and dD are projected axially and azimuthally to get the thrust dF and torque dQ contributions in an annular section; integration along the radius yields the thrust obtained and torque needed.

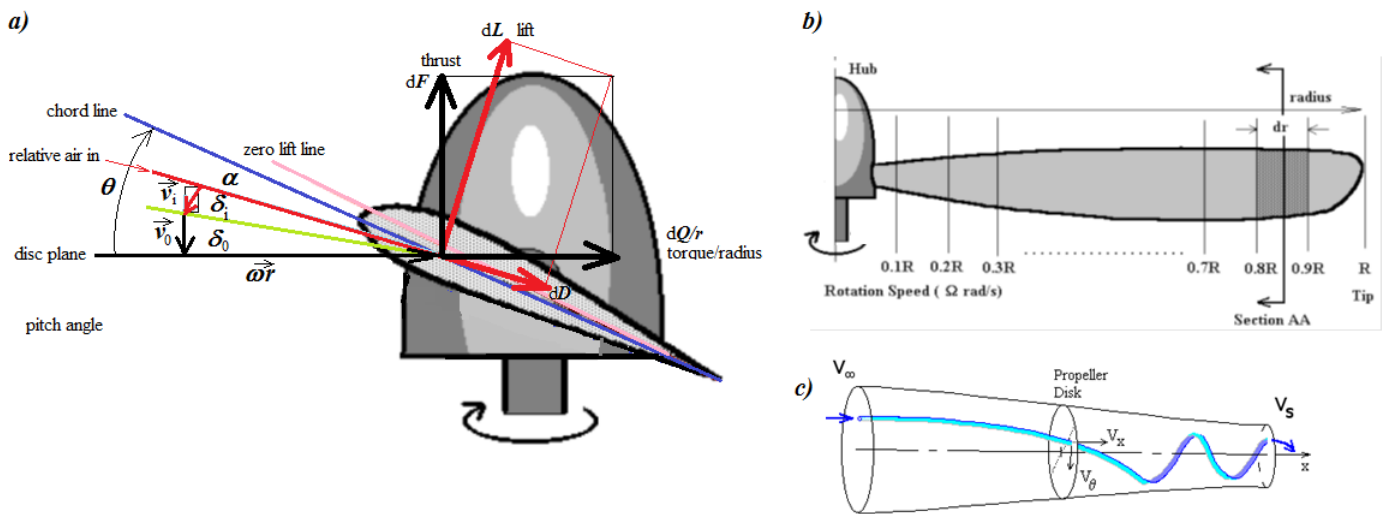


Fig. 4. Blade element model (BEM): a) view along the blade axis, with notation for velocities and forces, b) full blade view, c) propeller side view showing flow swirling ([Cambridge-MIT Institute](#)).

In axes fixed to the blade element, the angle of attack α (angle between the apparent wind direction and the chord line, see Fig. 4a) is found taking account of all relative velocities: rotational velocity $\omega r \vec{u}_\theta$ (\vec{u}_θ is the unitary azimuthal vector), propeller forward velocity $v_0 \vec{u}_z$ (\vec{u}_z is the unitary axial vector), and propeller-induced velocity $\vec{v}_i = a v_0 \vec{u}_z + b \omega r \vec{u}_\theta$ (with a and b as introduced above, in the actuator disc model); i.e. relative to the blade section, the apparent wind has the velocity $\vec{v}_{\text{wind}} = \vec{\omega} r + \vec{v}_0 + \vec{v}_i$ (Fig. 4a). Notice that the angle of attack, α , which for a non-advancing non-inducing propeller would be the pitch

angle θ , gets decreased by the effect of the advancing speed (δ_0 angle in Fig. 4a), and the effect of the induced speed (δ_i), with:

$$\delta_0 = \arctan \frac{v_0}{\omega r}, \quad \delta_i = \arctan \frac{v_i}{\sqrt{\omega^2 r^2 + v_0^2}}, \quad \alpha = \theta - \delta_0 - \delta_i \quad (5)$$

Once the angle of attack known (the induced speed is unknown, but being small, we will find it by iterations, starting with $a=b=0$), the aerofoil data (assumed normalised and tabulated) gives the annular contribution to thrust and torque:

$$\left. \begin{aligned} dL &= N_B c_L \frac{1}{2} \rho v_{\text{wind}}^2 c dr \\ dD &= N_B c_D \frac{1}{2} \rho v_{\text{wind}}^2 c dr \end{aligned} \right\} \rightarrow \begin{cases} dF = dL \cos(\theta - \alpha) - dD \sin(\theta - \alpha) \\ \frac{dQ}{r} = dL \sin(\theta - \alpha) + dD \cos(\theta - \alpha) \end{cases} \quad (6)$$

where N_B is the number of blades. But, from the momentum analysis performed under the disc actuator model, we get:

$$a(1+a) = \frac{dF}{4\pi r \rho v_0^2 dr}, \quad b = \frac{dQ}{4\pi r^3 \rho v_0 \omega (1+a) dr} \quad (7)$$

which allow us to compute a more precise value for the two components of the induced velocity and perform a new iteration until discrepancies become negligible, and then integrate radially to get the whole thrust and torque values.

Mind that we have not considered the influence of vortices shed from the blade tips (a tip loss correction factor was first developed by Prandtl).

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