NOZZLES

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A nozzle (from nose, meaning 'small spout') is a tube of varying cross-sectional area (usually axisymmetric) aiming at increasing the speed of an outflow, and controlling its direction and shape. Nozzle flow always generates forces associated to the change in flow momentum, as we can feel by hand-holding a hose and opening the tap. In the simplest case of a rocket nozzle, relative motion is created by ejecting mass from a chamber backwards through the nozzle, with the reaction forces acting mainly on the opposite chamber wall, with a small contribution from nozzle walls. As important as the propeller is to shaft-engine propulsions, so it is the nozzle to jet propulsion, since it is in the nozzle that thermal energy (or any other kind of high-pressure energy source) transforms into kinetic energy of the exhaust, and its associated linear momentum producing thrust.

The flow in a nozzle is very rapid (and thus adiabatic to a first approximation), and with very little frictional loses (because the flow is nearly one-dimensional, with a favourable pressure gradient except if shock waves form, and nozzles are relatively short), so that the isentropic model all along the nozzle is good enough for preliminary design. The nozzle is said to begin where the chamber diameter begins to decrease (by the way, we assume the nozzle is axisymmetric, i.e. with circular cross-sections, in spite that rectangular cross-sections, said two-dimensional nozzles, are sometimes used, particularly for their ease of directionability). The meridian nozzle shape is irrelevant with the 1D isentropic model; the flow is only dependent on cross-section area ratios.

Real nozzle flow departs from ideal (isentropic) flow on two aspects:

- Non-adiabatic effects. There is a kind of heat addition by non-equilibrium radical-species recombination, and a heat removal by cooling the walls to keep the strength of materials in long-duration rockets (e.g. operating temperature of cryogenic SR-25 rockets used in Space Shuttle is 3250 K, above steel vaporization temperature of 3100 K, not just melting, at 1700 K). Short-duration rockets (e.g. solid rockets) are not actively cooled but rely on ablation; however, the nozzle-throat diameter cannot let widen too much, and reinforced materials (e.g. carbon, silica) are used in the throat region.

- There is viscous dissipation within the boundary layer, and erosion of the walls, what can be critical if the erosion widens the throat cross-section, greatly reducing exit-area ratio and consequently thrust.
- Axial exit speed is lower than calculated with the one-dimensional exit speed, when radial outflow is accounted for.

We do not consider too small nozzles, say with chamber size <10 mm and neck size <1 mm, where the effect of boundary layers become predominant.

Restricting the analysis to isentropic flows, the minimum set of input parameters to define the propulsive properties of a nozzle (the thrust is the mass-flow-rate times the exit speed, $F = \dot{m}v_{\mathrm{e}}$) are:

- Nozzle size, given by the exit area, A_{e}; the actual area law, provided the entry area is large enough that the entry speed can be neglected, only modifies the flow inside the nozzle, but not the exit conditions.
- Type of gas, defined with two independent properties for a perfect-gas model, that we take as the thermal capacity ratio $\gamma \equiv c_p/c_v$, and the gas constant, $R \equiv R_{\mathrm{u}}/M$, and with R_{u}=8.314 J/(mol·K) and M being the molar mass, which we avoid using, to reserve the symbol M for the Mach number. If c_p is given instead of γ, then we compute it from $\gamma \equiv c_p/c_v = c_p/(c_p - R)$, having used Mayer's relation, $c_p - c_v = R$.
- Chamber (or entry) conditions: p_{c} and T_{c} (a relatively large chamber cross-section, and negligible speed, is assumed at the nozzle entry: $A_{\mathrm{c}} \to \infty$, $M_{\mathrm{c}} \to 0$). Instead of subscript 'c' for chamber conditions, we will use 't' for total values because the energy conservation implies that total temperature is invariant along the nozzle flow, and the non-dissipative assumption implies that total pressure is also invariant, i.e. $T_{\mathrm{t}} = T_{\mathrm{c}}$ and $p_{\mathrm{t}} = p_{\mathrm{c}}$.
- Discharge conditions: p_0, i.e. the environmental pressure (or back pressure), is the only variable of importance (because pressure waves propagate at the local speed of sound and quickly tend to force mechanical equilibrium, whereas the environmental temperature T_0 propagates by much slower heat-transfer physical mechanisms). Do not confuse discharge pressure, p_0, with exit pressure, p_{e}, explained below.

The objective is to find the flow conditions at the exit [p_{e},T_{e},v_{e}] for a given set of the above parameters, [A_{e},γ ,R ,p_{c},T_{c},,p_0], so that:

$$\dot{m} = \rho_{\mathrm{e}} v_{\mathrm{e}} A_{\mathrm{e}} = \frac{p_{\mathrm{e}}}{RT_{\mathrm{e}}} v_{\mathrm{e}} A_{\mathrm{e}}, \quad F = \dot{m} v_{\mathrm{e}}, \quad M_{\mathrm{e}} = \frac{v_{\mathrm{e}}}{\sqrt{\gamma R T_{\mathrm{e}}}} \qquad (1)$$

If the nozzle flow is subsonic, then the exit pressure coincides with the discharge pressure, $p_{\mathrm{e}} = p_0$, at the steady state (if at an initial state they were not equal, the time it would take to equalise is of the order of the nozzle length divided by the sound speed), and the other variables would be obtained from the isentropic relations, i.e.:

$$p_e = p_0, \quad \left(\frac{p_t}{p_e}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_t}{T_e} = 1 + \frac{v_e^2}{2c_pT_e} = 1 + \frac{\gamma-1}{2}M_e^2 \tag{2}$$

Converging nozzles are used to accelerate the fluid in subsonic gas streams (and in liquid jets), since at low speeds density do not vary too much, and $\dot{m} = \rho v A = \text{const}$ can be approximated by vA=const. Liquid jets and low speed gas flows can be studied with classical Bernoulli equation (until cavitation effects appear in liquid flows), but high-speed gas dynamics is dominated by compressibility effects in the liquid. By the way, we do not considered here multiphase flow in nozzles. But when the flow is supersonic at some stage (even just at the exit), $p_e \neq p_0$, and a more detailed analysis is required. Before developing it, let summarise the results.

A converging nozzle can only become supersonic at the exit stage; the speed increases monotonically along the nozzle. If a converging nozzle is fed from a constant pressure constant temperature chamber, the flow rate grows as the discharge pressure is being reduced, until the flow becomes sonic (choked) and the flow rate no longer changes with further decreasing in discharge-pressure (a set of expansion waves adjust the exit pressure to this lower discharge pressure). Except for old-time turbojets and military fighter aircraft, all commercial jet engines (after Concorde was retired) use converging nozzles discharging at subsonic speed (both, the hot core stream and the colder fan stream).

A converging-diverging nozzle ('condi' nozzle, or CD-nozzle), is the only one to get supersonic flows with M>1 (when chocked). It was developed by Swedish inventor Gustaf de Laval in 1888 for use on a steam turbine. Supersonic flow in CD-nozzles presents a rich behaviour, with shock waves and expansion waves usually taking place inside and/or outside. Several nozzle geometries have been used in propulsion systems:

1. The classical quasi-one-dimensional Laval nozzle, which has a slender geometry, with a rapidly converging short entrance, a rounded throat, and a long conical exhaust of some 15º half-cone angle (the loss of thrust due to jet divergence is about 1.7%). Rarely used in modern rockets.
2. Bell-shape nozzles (or parabolic nozzles), which are as efficiency as the simplest conical nozzle, but shorter and lighter, though more expensive to manufacture. They are the present standard in rockets; e.g. the Shuttle main engine (SME) nozzles yield 99% of the ideal nozzle thrust (and the remainder is because of wall friction, not because of wall shape effect).
3. Annular and linear nozzles, designed to compensate ambient pressure variation, like the Aerospike nozzle. They are under development.

We present below the 1D model of gas flow in nozzles. For more realistic design, beyond this simple model, a 2D (or axisymmetric) analysis by the method of characteristics and boundary layer effects should follow, to be completed with a full 3D nozzle-flow analysis by CFD.

NOZZLE FLOW EQUATIONS

Let us consider the steady isentropic 1D gas dynamics in a CD-nozzle, with the perfect gas model (i.e. $pV=mRT$ and, taking T=0 K as energy reference, $h=c_pT$). Conservation of mass, momentum, and energy, in terms of the Mach number, $M\equiv v/c$ (where $c=\sqrt{\gamma RT}$ stands for the sound speed), become:

$$\dot{m}=\rho vA=\text{const}=\frac{p}{RT}\left(M\sqrt{\gamma RT}\right)A \quad\rightarrow\quad \frac{dp}{p}-\frac{dT}{2T}+\frac{dM}{M}+\frac{dA}{A}=0 \tag{3}$$

$$\rho v dv=-dp\rightarrow\frac{dv^2}{2}+\frac{dp}{\rho}=0\xrightarrow{dh=Tds+vdp}\frac{dv^2}{2}+dh-Tds=0\xrightarrow{h_t=h+\frac{v^2}{2}=\text{const}}ds=0 \tag{4}$$

$$h_t=h+\frac{v^2}{2}=\text{const}=c_pT+\frac{1}{2}M^2\gamma RT \quad\rightarrow\quad \frac{dT}{T}\left(1+\frac{\gamma-1}{2}M^2\right)+\left(\gamma-1\right)MdM=0 \tag{5}$$

where logarithmic differentiation has been performed. Notice that, with this model, the isentropic condition can replace the momentum equation, so that differentiation of the isentropic relations for a perfect gas $T/p^{(\gamma-1)/\gamma}$=const, yields:

$$\frac{dT}{T}=\frac{\gamma-1}{\gamma}\frac{dp}{p} \tag{6}$$

The energy balance ($\Delta h_t=q+w$) implies the conservation of total enthalpy and total temperature ($h_t=c_pT_t$), and the non-friction assumption implies the conservation of total pressure (p_t), with the relations between total and static values given by:

$$\frac{T_t}{T}=1+\frac{v^2}{2c_pT}=1+\frac{\gamma-1}{2}M^2=\left(\frac{p_t}{p}\right)^{\frac{\gamma-1}{\gamma}} \tag{7}$$

Notice that, with the perfect gas model, γ remains constant throughout the expansion process. However, when the engine flow is composed of hot combustion products, real gas effects become important, and as the gas expands, γ shifts as a result of changes in temperature and in chemical composition. Maximum thrust is obtained if the gas composition is in chemical equilibrium throughout the entire nozzle expansion process.

Choosing the cross-section area of the duct, A, as independent variable, the variation of the other variables can be explicitly found from (3)-(6) to be:

$$\left(1-M^2\right)\frac{dT}{T}=\left(\gamma-1\right)M^2\frac{dA}{A} \tag{8}$$

$$\left(1-M^2\right)\frac{dp}{p}=\gamma M^2\frac{dA}{A} \tag{9}$$

$$\left(1-M^2\right)\frac{dM}{M}=-\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dA}{A} \tag{10}$$

$$\left(1-M^2\right)\frac{dv}{v}=-\frac{dA}{A} \tag{11}$$

Equations (8)-(11) show that:

- In converging sections ($dA<0$):
  - When the flow is subsonic ($M<1\rightarrow(1-M^2)>0$): speed increases ($dv>0$), Mach-number increases ($dM>0$), but pressure and temperature decrease.
  - When the flow is supersonic ($M>1\rightarrow(1-M^2)<0$): speed decreases ($dv<0$), Mach-number decreases ($dM<0$), but pressure and temperature increase.
- In diverging sections ($dA>0$):
  - When the flow is subsonic ($M<1\rightarrow(1-M^2)>0$): speed decreases ($dv<0$), Mach-number decreases ($dM<0$), but pressure and temperature increase.
  - When the flow is supersonic ($M>1\rightarrow(1-M^2)<0$): speed increases ($dv>0$), Mach-number increases ($dM>0$), but pressure and temperature decrease.

Choked flow

Choking is a compressible flow effect that obstructs the flow, setting a limit to fluid velocity because the flow becomes supersonic and perturbations cannot move upstream; in gas flow, choking takes place when a subsonic flow reaches M=1, whereas in liquid flow, choking takes place when an almost incompressible flow reaches the vapour pressure (of the main liquid or of a solute), and bubbles appear, with the flow suddenly jumping to M>1.

Going on with gas flow and leaving liquid flow aside, we may notice that M=1 can only occur in a nozzle neck, either in a smooth throat where dA=0, or in a singular throat with discontinuous area slope (a kink in nozzle profile, or the end of a nozzle). Naming with a '*' variables the stage where M=1 (i.e. the sonic section, which may be a real throat within the nozzle or at some extrapolated imaginary throat downstream of a subsonic nozzle), and integrating from A to A^*, equations (8)-(10) become:

$$\frac{T^*}{T}=\frac{1+\frac{\gamma-1}{2}M^2}{\frac{\gamma+1}{2}}\xrightarrow{T_t=T\left(1+\frac{\gamma-1}{2}M^2\right)}\frac{T^*}{T_t}=\frac{2}{\gamma+1} \qquad (12)$$

$$\frac{p^*}{p}=\left(\frac{1+\frac{\gamma-1}{2}M^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma}{\gamma-1}}\xrightarrow{\frac{T}{T_t}=\left(\frac{p}{p_t}\right)^{\frac{\gamma-1}{\gamma}}}\frac{p^*}{p_t}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \qquad (13)$$

$$\frac{A^*}{A}=M\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (14)$$

where the expressions for total temperature T_t and total pressure p_t has been substituted to show that temperature and pressure at the throat (also known as critical values), are just a function of γ, since, for isentropic flows, total conditions do not change along the stream.

Although the equations above apply to all 1D isentropic perfect-gas flows, equations (12)-(14) make use of conditions at M=1 (real or virtual), and it is worth analysing special cases in particular detail.

Area ratio

Nozzle area ratio ε (or nozzle expansion ratio) is defined as nozzle exit area divided by throat area, $\varepsilon \equiv A_e/A^*$, in converging-diverging nozzles, or divided by entry area in converging nozzles. Notice that ε so defined is ε>1, but sometimes the inverse is also named 'area ratio' (this contraction area ratio is bounded between 0 and 1); however, although no confusion is possible when quoting a value (if it is >1 refers to A_e/A^*, and if it is <1 refers to A^*/A_e), one must be explicit when saying 'increasing area ratio' (we keep to $\varepsilon \equiv A_e/A^*>1$).

To see the effect of area ratio on Mach number, (14) is plotted in Fig. 1 for ideal monoatomic (γ=5/3), diatomic (γ=7/5=1.40), and low-gamma gases as those of hot rocket exhaust (γ=1.20); gases like CO_2 and H_2O have intermediate values (γ=1.3). Notice that, to get the same high Mach number, e.g. M=3, the area ratio needed is A^*/A=0.33 for γ=1.67 and A^*/A=0.15 for γ=1.20, i.e. more than double exit area for the same throat area (that is why supersonic wind tunnels often use a monoatomic working gas.

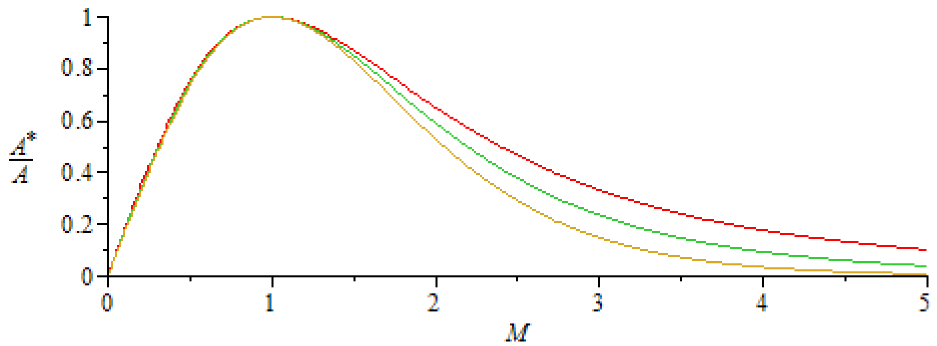


Fig. 1. Ratio A^*/A (i.e. throat area divided by local area) vs. Mach number M, for γ=1.20 (beige), γ=1.40 (green), and γ=1.67] (red).

Exercise 1. A steam flow of 0.1 kg/s expands isentropically in a nozzle from a chamber at 300 kPa and 300 ºC to an outside atmosphere at 100 kPa. Find:

a) If the nozzle is converging or converging-diverging, and the exit Mach number.
b) Exit area and minimum area.

Sol.: a) For steam, γ=1.30 (e.g. from c_p=2050 J/(kg·K), $\gamma \equiv c_p/c_v = c_p/(c_p-R)$=2050/(2050-462)=1.29). For isentropic nozzles, p_t=const and T_t=constant. Choking must occur if $p_e/p_t<(2/(\gamma+1))^{\gamma/(\gamma-1)}$=0.55 and in our case is p_e/p_t=100/300=0.33; hence, it is a CD-nozzle. Exit temperature is $T_e=T_t(p_e/p_t)^{(\gamma-1)/\gamma}$=445 K (172 ºC). Solving in (13) one finds p^*=164 kPa and M_e=1.39.

b) Exit area can be found from (3) since p, T and M are known, obtaining A_e=2.87 cm^2. Throat area can be obtained from (14) with A_e=2.87 cm^2 and M_e=1.39, with a result A^*=2.57 cm^2. Exit speed is $v_e = M_e\sqrt{\gamma R T_e}$ =717 m/s.

Exercise 2. A flow of 100 kg/s of exhaust gases expands in a nozzle with 0.95 isentropic efficiency from a low-speed entrance at 300 kPa and 400 ºC to an outside atmosphere at 100 kPa. Assuming as averaged values c_p=1100 J/(kg·K) and γ=1.35, find:

a) Exit area, minimum area, and exhaust Mach number and speed, assuming isentropic flow.
b) Corrections due to the stated efficiency.

Sol.: a) Choking must occur if $p_e/p_t<(2/(\gamma+1))^{\gamma/(\gamma-1)}=0.54$ and in our case is $p_e/p_t=100/300=0.33$; hence, it is a CD-nozzle. For isentropic flow, exit temperature is $T_e=T_t(p_e/p_t)^{(\gamma-1)/\gamma}=506$ K, and exit speed $v_e=\sqrt{2c_p(T_t-T_e)}=606$ m/s, with Mach number $M_e=v_e/\sqrt{\gamma RT_e}=1.37$. Exit area can be found from (3) since p_e, T_e and M_e are known, obtaining $A_e=0.238$ m^2. Throat area can be obtained from (14) with $A_e=0.238$ m^2 and $M_e=1.37$, with a result $A^*=0.216$ m^2.

b) From the definition of isentropic nozzle efficiency, $\eta\equiv(h_t-h_e)/(h_t-h_{es})=v_e^2/v_{es}^2$, with the isentropic values found above, $T_{es}=506$ K and $v_{es}=606$ m/s, we get $T_{es}=515$ K and $v_{es}=590$ m/s, with Mach number $M_e=v_e/\sqrt{\gamma RT_e}=1.33$. Exit area can be found from (3) since p_e, T_e and M_e are known, obtaining $A_e=0.249$ m^2. Throat area should be assumed to coincide with the isentropic value, $A^*=0.216$ m^2, since viscous dissipation in the converging section would be a small fraction of the total dissipation.

Converging nozzle

In a converging nozzle, cross-section area smoothly decreases from a larger value (usually assumed a plenum chamber with $M\to 0$, $p_c=p_t$) to a smaller value (exit section A_e, with M_e and p_e). The mass flow rate in terms of static or total conditions at any stage, with the isentropic relations (7), is:

$$\dot{m}=\rho vA=\sqrt{\frac{\gamma}{R}}\frac{p}{\sqrt{T}}MA=\sqrt{\frac{\gamma}{R}}\frac{p_t}{\sqrt{T_t}}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}MA \qquad (15)$$

with $\dot{m}$=const, T_t=const, p_t=const. Whatever the area law, the flow accelerates to a maximum speed at the exit. Two cases may appear:

- Subsonic exit ($M_e<1$).
- Sonic exit ($M_e=1$).

For subsonic exit, exit pressure equals ambient pressure ($p_e=p_0$), and exit conditions are:

$$p_e=p_0,\quad T_e=T_t\left(\frac{p_e}{p_t}\right)^{\frac{\gamma-1}{\gamma}},\quad M_e=\sqrt{\frac{2}{\gamma-1}\left(\left(\frac{p_t}{p_e}\right)^{\frac{\gamma-1}{\gamma}}-1\right)},\quad \dot{m}=\sqrt{\frac{\gamma}{R}}\frac{p_t}{\sqrt{T_t}}\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}M_eA_e \qquad (16)$$

valid only if $M_e\le 1$. The limit $M_e=1$ (choking conditions) will be reached when:

$$\frac{p_e}{p_t}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}},\quad \frac{T_e}{T_t}=\frac{2}{\gamma+1},\quad M_e=1,\quad \dot{m}=\sqrt{\frac{\gamma}{R}}\frac{p_t}{\sqrt{T_t}}\left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}A_e=\frac{\gamma p_t}{c_c}\left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}A_e \qquad (17)$$

where $c_c=\sqrt{\gamma RT_t}$ is the sound speed at chamber conditions. In conclusion, if, for given entry conditions (p_t and T_t), ambient pressure is being lowered from the no-flow condition, $p_0=p_t$, first a subsonic flow develops, until $p_0=p^*=p_t(2/(\gamma+1))^{\gamma/(\gamma-1)}$, e.g. $p_0/p_t=0.53$ for $\gamma=1.4$, where mass flow rate is at a maximum, and a further decrease of ambient pressure has no effect in nozzle flow (no pressure-information can go upstream); a fan of oblique supersonic expansion waves appears just at the exit, to accommodate exit pressure p_e (fixed at (17) value) to ambient pressure $p_0<p_e$, with a bulging of the exhaust jet.

But nozzle flow becomes choked if $p_0/p_t<p^*/p_t=(2/(\gamma+1))^{\gamma/(\gamma-1)}$, e.g. if $p_0/p_t<0.53$ for $\gamma=1.4$. This is the typical case of a high-pressure chamber discharging through hole, since, unless the hole is a well-designed converging-diverging nozzle, the flow will separate at the maximum constriction (the throat), and will behave as a converging nozzle. If the feeding chamber is at a steady state (i.e. T_t=const, p_t=const), then the choked flow is invariant, and the mass-flow-rate a constant, (17), for given exit area A_e, no matter how much the discharge pressure is lowered. But, if the feeding chamber is unsteady, e.g. depressurising because of the escaping mass, then, even if the nozzle remains choked, the mass flow rate, given by (17), decreases with time, with the following invariant:

$$\frac{\dot{m}\sqrt{T_t}}{p_t}=\text{const} \tag{18}$$

i.e. $\dot{m}$ changes with changing entry conditions. The two extreme cases of discharge from a gas tank are: isothermal (T_t=const, so that $\dot{m}\propto p_c$), and adiabatic (isentropic, if internal dissipation is negligible, i.e. $T_t=\text{const}\cdot p_t^{(\gamma-1)/\gamma}$, so that $\dot{m}\propto p_c^{(\gamma+1)/(2\gamma)}$, e.g. for air with $\gamma=1.4$, $\dot{m}\propto p_c^{0.857}$). Notice that a gas tank discharges more slowly if thermally isolated than if kept isothermal (in the latter, heat addition tends to increase pressure and help the ejection).

Exercise 3. Consider a habitable spacecraft module of 50 m^3 with air at 100 kPa and 300 K, and find the time it would take to depressurize through a 1 cm^2 hole.

Sol.: The air inside escapes through an area $A_e=10^{-4}$ m^2 (we disregard the local details of the hole, which may typically have a discharge coefficient $c_d=0.9$ (reducing the effective area accordingly). The discharge is choked all time since external pressure is $p_e\approx 0$, and thus $M_e=1$. We further assume that the small escape area makes the process slow enough to be approximated as isothermal. A first estimation is that $\dot{m}=\rho_e v_e A_e\approx\rho_c c_c A_e=1\cdot 300\cdot 10^{-4}=0.03$ kg/s, so that the initial 50 kg of air would escape in about 50/0.03=1700 s.

More precisely, the mass flow rate is given by (17), $\dot{m}=f(\gamma)p_t A_e/c_c$, with $f(\gamma)=0.805$ for air ($\gamma=1.4$), $c_c=\sqrt{\gamma R T_t}$=347 m/s, and pressure is proportional to mass, $p_t=mRT/V$. In terms of pressure, the mass balance $\mathrm{d}m/\mathrm{d}t=-\dot{m}$ yields:

$$\frac{V}{RT}\frac{\mathrm{d}p_t}{\mathrm{d}t}=-\dot{m}=-\frac{f(\gamma)p_t A_e}{c_c}\quad\rightarrow\quad\frac{\mathrm{d}p_t}{p_t}=\frac{-f(\gamma)RTA_e}{c_c V}\mathrm{d}t\quad\rightarrow\quad p_t=p_{t1}\exp\left(\frac{-f(\gamma)RTA_e t}{c_c V}\right)$$

i.e. pressure decreases exponentially to 0 with an initial rate of 40 Pa/s, i.e. with a characteristic time $c_cV/(f(\gamma)RTA_e=(V/(c_cA_e))f(\gamma)/\gamma=2500$ s; this is the time it takes for $p_t/p_{t1}=1/e=0.37$; consciousness is lost in 3 to 6 minutes at this pressure level, where oxygen partial pressure is 7.8 kPa).

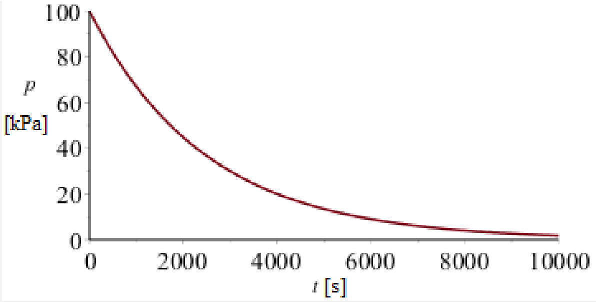


Fig. E3. Depressurization of a habitable spacecraft module.

If the depressurization were adiabatic, pressure loss would slow down slightly due to the large temperature drop (e.g., after 2000 s, instead of 45 kPa given by the previous model, pressure would be 47 kPa, but temperature would had fall to 241 K.

Converging-diverging nozzle

A converging-diverging nozzle ('condi' nozzle, or CD-nozzle) must have a smooth area law, with a smooth throat, $\mathrm{d}A/\mathrm{d}x=0$, for the flow to remain attached to the walls. The flow starts from rest and accelerates subsonically to a maximum speed at the throat, where it may arrive at $M<1$ or at $M=1$, as for converging nozzles. Again, for the entry conditions we use 'c' (for chamber) or 't' (for total), we use 'e' for the exit conditions, and '*' for the throat conditions when it is choked ($M^*=1$).

If the flow is subsonic at the throat, it is subsonic all along the nozzle, and exit pressure p_e naturally adapts to environmental pressure p_0 because pressure-waves travel upstream faster (at the speed of sound) than the flow (subsonic), so that $p_e/p_0=1$. But now the minimum exit pressure for subsonic flow is no longer $p_e=p_t(2/(\gamma+1))^{\gamma/(\gamma-1)}$ ($p_e/p_0=0.53$ for $\gamma=1.4$), since the choking does not take place at the exit but at the throat, i.e. it is the throat condition that remains valid, $p^*=p_t(2/(\gamma+1))^{\gamma/(\gamma-1)}$, e.g. $p^*/p_0=0.53$ for $\gamma=1.4$; now the limit for subsonic flow is $p_{e,min,sub}>p^*$ because of the pressure recovery in the diverging part. However, if the flow is isentropic all along the nozzle, be it fully subsonic or supersonic from the throat, the isentropic equations apply:

$$\left(\frac{p_t}{p_e}\right)^{\frac{\gamma-1}{\gamma}}=\frac{T_t}{T_e}=1+\frac{\gamma-1}{2}M_e^2,\quad M_e=\sqrt{\frac{2}{\gamma-1}\left(\left(\frac{p_t}{p_e}\right)^{\frac{\gamma-1}{\gamma}}-1\right)}$$
$$\dot{m}=\rho_e v_e A_e=\frac{\gamma p_t A_e}{\sqrt{\gamma R T_t}}M_e\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{2-\gamma}{2}} \qquad (19)$$

But if the flow gets sonic at the throat, several downstream conditions may appear. The control parameter is discharge pressure, p_0. Let consider a fix-geometry CD-nozzle, discharging a given gas from a reservoir with constant conditions (p_t,T_t). When lowering the environmental pressure, p_0, from the no flow conditions, $p_0=p_t$, we may have the following flow regimes (a plot of pressure variation along the nozzle is sketched in Fig. 2):

- Subsonic throat, implying subsonic flow all along to the exit (evolution *a* in Fig 2).
- Sonic throat (no further increase in mass-flow-rate whatever low the discharge pressure let be).
    - Fully subsonic flow except at the throat (evolution *b*).

- Flow becomes supersonic after the throat, but, before exit, a normal shockwave causes a sudden transition to subsonic flow (evolution *c*). It may happen that the flow detaches from the wall (see the corresponding sketch).
- Flow becomes supersonic after the throat, with the normal shockwave just at the exit section (evolution *d*).
- Flow becomes supersonic after the throat, and remains supersonic until de exit, but there, three cases may be distinguished:
  - Oblique shock-waves appear at the exit, to compress the exhaust to the higher back pressure (evolution *e*). The types of flow with shock-waves (*c*, *d* and *e* in Fig. 2) are named 'over-expanded' because the supersonic flow in the diverging part of the nozzle has lowered pressure so much that a recompression is required to match the discharge pressure. That is the normal situation for a nozzle working at low altitudes (assuming it is adapted at higher altitudes); it also occurs at short times after ignition, when chamber pressure is not high enough.
  - Adapted nozzle, where exit pressure equals discharge pressure (evolution *f*). Notice that, as exit pressure p_e only depends on chamber conditions for a choked nozzle, a fix-geometry nozzle can only work adapted at a certain altitude (such that $p_0(z)=p_e$).
  - Expansion waves appear at the exit, to expand the exhaust to the lower back pressure (evolution *e*); this is the normal situation for nozzles working under vacuum. This type of flow is named 'under-expanded' because exit pressure is not low-enough, and additional expansion takes place after exhaust.

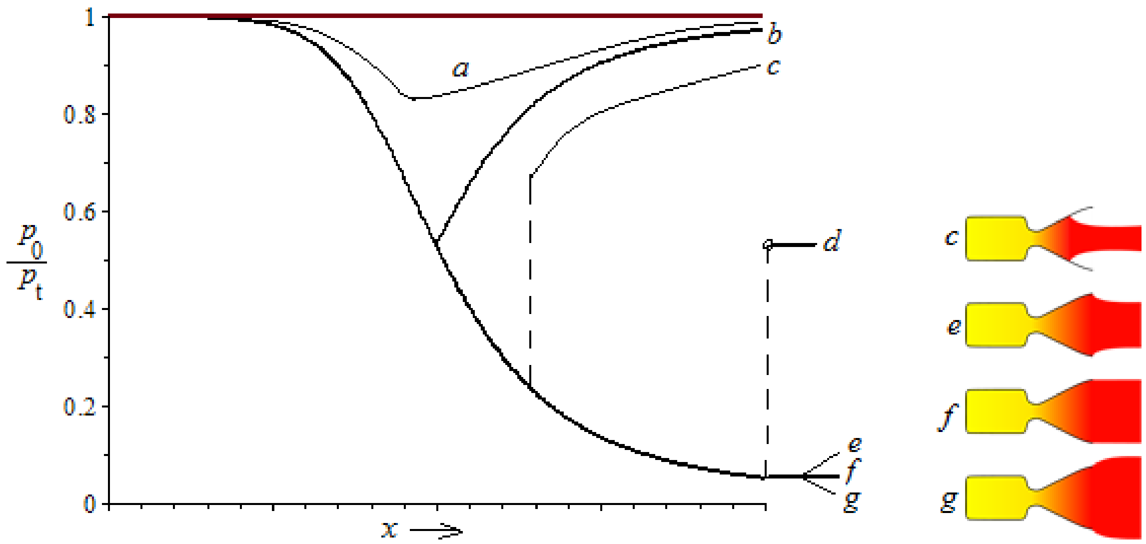


Fig. 2. Nozzle flows for constant entry conditions (p_t,T_t), as a function of discharge pressure p_0. As p_0 is being decreased, the flow starts being subsonic (*a*) all along the nozzle length (*x*), then it becomes choked (and the flow no longer changes in the converging part). But the flow in the diverging part may be subsonic (*b*), or a transition from supersonic to subsonic occur within (*c*, from *b* to *d*), or the supersonic flow at the exit be followed by compression waves (*e*), be adapted (*f*), or be followed by expansion waves (*g*); the latter case is said under-expanded, and is typical under vacuum.

In the converging-diverging nozzle used in supersonic aircraft, both the throat area and the exit area should be optimised for maximum thrust as a function of altitude and flight speed, but in practice there is a single mechanical adjustment, using petals to achieve a variable area nozzle. In rockets, fix-area nozzles is the rule.

When the flow is isentropic all along the nozzle, i.e. for values of p_0/p_t from d to g in Fig. 2, the exit Mach number M_e is given by:

$$\frac{A^*}{A_e} = M_e \left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_e^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{20}$$

Mind that, solving (20) for M yields two solutions, $M_{e,sub}<1$ and $M_{e,sup}>1$, corresponding to the isentropic exit pressure and temperature pairs:

$$\left(\frac{p_t}{p_{e,sub}} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_t}{T_{e,sub}} = 1+\frac{\gamma-1}{2}M_{e,sub}^2 \tag{21}$$

$$\left(\frac{p_t}{p_{e,sup}} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_t}{T_{e,sup}} = 1+\frac{\gamma-1}{2}M_{e,sup}^2 \tag{22}$$

The supersonic mass-flow-rate and exit speed in the isentropic discharge through a nozzle are:

$$\dot{m} = \rho^* v^* A^* = \frac{p^*}{RT^*}\sqrt{\gamma RT^*}A^* = \frac{\gamma p^* A^*}{\sqrt{\gamma RT^*}} = \frac{\gamma p_t A^*}{\sqrt{\gamma RT_t}}\left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{23}$$

$$v_e = \sqrt{\frac{2\gamma RT_t}{\gamma-1}\left[1-\left(\frac{p_e}{p_t} \right)^{\frac{\gamma-1}{\gamma}} \right]} = \sqrt{\frac{2\gamma RT_t}{\gamma-1}\left[1-\frac{1}{1+\frac{\gamma-1}{2}M_e^2} \right]} \tag{24}$$

Notice that, although it is often said that $\dot{m}$ is constant in a choked nozzle (critical flow-rate), what is meant is that the mass-flow-rate does not depend on back pressure (provided the flow becomes supersonic), but $\dot{m}$ is almost proportional to chamber pressure (and depends on temperature and gas properties too; though they are almost invariable during normal operation of rockets; see Chamber pressure equation, below).

Sometimes, a characteristic speed v_c is defined as chamber pressure (p_t) times throat area (A^*) divided by mass flow rate ($\dot{m}$), i.e. by $v_c \equiv p_t A^*/\dot{m}$, a modified sound speed independent of the exit area, as can be deduced by substitution from (23); when using such a characteristic speed, a non-dimensional thrust coefficient is defined by $c_F \equiv F/(p_t A^*) = v_e/v_c + (A_e/A^*)(p_e - p_0)/p_t$, such that for an adapted nozzle it is $c_F = v_e/v_c$.

Notice also that the maximum exit speed corresponds to an infinite expansion where all the thermal energy goes to kinetic energy, is $c_p(T_t-T_e)=c_pT_t=v_e^2/2$; e.g. for air at 288 K expanding isentropically to vacuum, $v_e=\sqrt{2c_pT_t}=\sqrt{2\cdot 1000\cdot 288}=760$ m/s. Some typical values of the exhaust gas velocity for rocket engines burning various propellants are:

- For solid propellants and for liquid monopropellants v_e=2000 to 3000 m/s; e.g. hydrazine catalytic decomposition at 7 MPa generating gases with average M=0.022 kg/mol and γ=1.22 at T_t=2500 K, would yield a maximum of $v_e=\sqrt{2c_pT_t}=\sqrt{2\cdot 2100\cdot 2500}=3200$ m/s under vacuum.
- For liquid bipropellants v_e=3000 to 4500 m/s; e.g. for a cryogenic H_2/O_2 rocket generating water vapour (and some dissociates) at 20 MPa and 3250 K (with excess of H_2), the maximum exit speed is $v_e=\sqrt{2c_pT_t}=\sqrt{2\cdot 2500\cdot 3500}=4200$ m/s.

An example of CD-nozzle flow computed with the above equations is shown in Fig. 3.

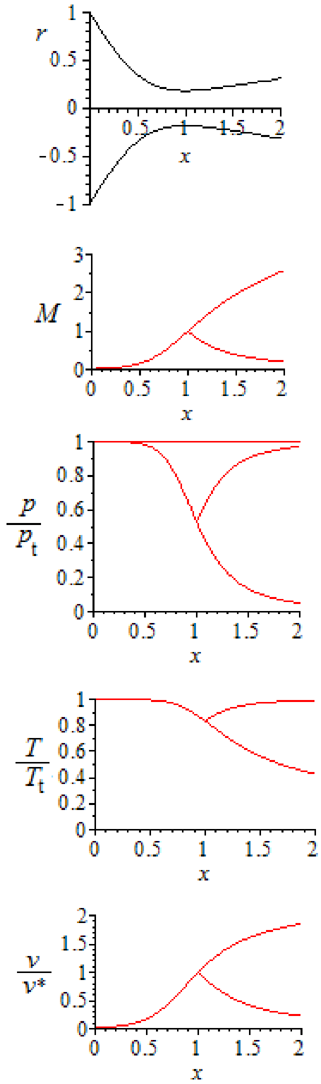


Fig. 3. A computed example of choked flow of air (γ=1.4) in the CD-nozzle geometry shown on top (nozzle throat at x=1; in scaled arbitrary dimensions, area ratio A_e/A^*=2.9). Two isentropic solutions exist: one totally subsonic (except at the throat, which is sonic), and another that becomes supersonic after the throat. The plots are: Mach number, local pressure relative to chamber value, local temperature relative to chamber value, and local speed relative to its throat value (speed of sound at throat conditions). More detailed nozzle flow simulations can be found aside.

Looking at the smooth rounding of the nozzle neck (Fig. 3a), it seems amazing the kink in all flow variables corresponding to the full subsonic solution (the explanation resides in the singularity that (8)-(11) have for M=1). Another astonishing result is the very rapid pressure changes at the neck: in the length from x=0.86 to x=1.18 in Fig. 3a, where the nozzle radius varies from r=0.19 to r=0.19 through r=0.18 at the throat (a 10 % in area change from neck to ends), pressure (scaled with the constant total pressure) varies from p/p_t=0.73 at x=0.86 to p/p_t=0.32 at x=1.18 if the flow becomes supersonic (or recedes to the same subsonic value, p/p_t=0.73 at x=1.18 if it remains subsonic), passing by p^*/p_t=0.53 at x=1; i.e. between the two sections with radius 5 % larger than the minimum, pressure decreases a 56% (in supersonic flow; or it recovers, after decreasing a 27%).

It is also impressive how soon the choking occurs when backpressure p_e is being decreased: at p_e/p_t=1 there is no flow, and at p_e/p_t=0.97 the nozzle is already choked, with a fix mass flow rate whatever the value of p_e/p_t<0.97 (if entry conditions are maintained). But, if the flow is choked, how it adapts to the changing exit pressure? Through discontinuities in the flow, breaking the isentropic condition.

Exercise 4. A liquid bipropellant rocket to be used for the second stage of a space launcher, works at 7 MPa and 3300 K in the combustion chamber, generating gases with a mean molar mass M=0.020 kg/mol, and γ=1.30, having a nozzle exit diameter of 1.5 m and nozzle expansion ratio of 50. Find:

a) The exit Mach number, assuming fully supersonic flow.
b) Discharge pressure condition for adapted nozzle.
c) Propellants flow rate.
d) Exit speed and thrust at 20 km altitude.

Sol.: a) Given the expansion ratio, $\varepsilon \equiv A_e/A^*$, equation (20) yields the exit Mach number:

$$\frac{A^*}{A_e}=\frac{1}{\varepsilon}=M_e\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_e^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \xrightarrow[\gamma=1.30]{\varepsilon=1/50} 0.02=M_e\frac{1.71}{\left(1+0.15M_e^2\right)^{3.83}} \rightarrow M_e=5.1$$

b) The condition is $p_0=p_{e,sup}$, where the isentropic exit pressure is obtained by (22):

$$\frac{p_t}{p_{e,sup}}=\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}} \xrightarrow[\substack{\gamma=1.30\\ M_e=5.1}]{p_t=7\text{ MPa}} p_{e,sup}=7.3\text{ kPa}$$

i.e. the nozzle would be adapted if the environment is at p_0=7.3 kPa (z=18.2 km altitude in ISA model); if p_0<7.3 kPa the flow is under-expanded (fully supersonic and with additional expansion waves at the exit); however, if p_0>7.3 kPa, the flow is over-expanded (and recompressed by shock waves, either at the exit stage, or within the nozzle).

c) The mass flow rate of propellants equals the mass flow rate of exhaust gases, which is obtained by (23) either at throat conditions or at the exit stage (it does not depend on p_0, provided the nozzle is choked):

$$\dot{m}=\rho_e v_e A_e=\frac{\gamma p_t A_e}{\sqrt{\gamma R T_t}}M_e\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{2-\gamma}{2}} \xrightarrow[\substack{A_e=\pi 1.5^2/4\\ R=8.3/0.020\\ T_t=3300\text{ K}\\ M=5.1}]{\substack{\gamma=1.30\\ p_t=7\text{ MPa}}} \dot{m}=141\text{ kg/s}$$

d) We have seen in b) that the flow is fully supersonic for p_0<7.3 kPa (z>18.2 km altitude in ISA model), so that the exit velocity is the same as for an adapted nozzle and can be obtained from (24):

$$v_e=\sqrt{\frac{2\gamma RT_t}{\gamma-1}\left[1-\left(\frac{p_e}{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right]}=\sqrt{\frac{2\gamma RT_t}{\gamma-1}\left[1-\frac{1}{1+\frac{\gamma-1}{2}M_e^2}\right]}\xrightarrow[M=5.1]{\gamma=1.30,\ R=8.3/0.020,\ T_t=3300\text{ K}}v_e=3070\text{ m/s}$$

Another way could have been through $\dot{m}=\rho_e v_e A_e=p_e v_e A_e/(RT_e)$ with p_e=7.3 kPa, $A_e=\pi\cdot1.5^2/4=1.77$ m^2, $R=R_u/M$=8.3/0.020=415 J/(kg·K), and $T_e=T_t/(1+(\gamma-1)M_e^2/2)$= 3300/(1+0.15·5.1^2)=677 K. The thrust F is:

$$F=\dot{m}v_e+(p_e-p_0)A_e=141\cdot3070+(7300-5530)\cdot1.77=433\text{ kN}+31\text{ kN}=464\text{ kN}$$

i.e. F=464 kN (433 kN from jet momentum, and 31 kN from excess pressure relative to the environment).

Discontinuities in nozzle flow: normal and oblique shocks, expansion fans. Mach diamonds

Consider the isentropic pressure evolution along the nozzle in Fig. 2, . Far downstream of the exit stage, the exhaust jet (say, more than a couple of exit diameters), the jet , leaving aside a possible adjustment close to the exit, must be sensibly equal the exit pressure

We have seen that, for given entry conditions, several cases of nozzle flow appear as a function of the imposed discharge pressure p_0, to which the exhaust jet must adapt, since a free jet cannot withstand transversal pressure gradients. This pressure-adaptation may be through small (linear) or strong (non-linear) pressure waves. By comparing relative exit pressure p_e/p_t, with relative back pressure p_0/p_t, the possible flow configurations are:

- If $1>p_e/p_t>p_{e,sub}/p_t$, then the matching $p_e=p_0$ is by acoustic waves travelling upstream from the exit to the entrance, and the flow is subsonic all along the nozzle length; for the example plotted in Fig. 3, $1>p_e/p_t>0.97$.
- If $p_e/p_t=p_{e,sub}/p_t$, the flow is choked but subsonic except at the neck (this is the limit of the case above).
- If $p_{e,sub}/p_t>p_e/p_t>p_{e,sup}/p_t$, then the matching $p_e=p_0$ is by acoustic waves travelling upstream from the exit to a section between exit and throat, where a strong shock takes place; the location of this normal shockwave develops some distance downstream (the further down the lower the backpressure), with subsonic flow beyond, matching the ambient backpressure. For the example plotted in Fig. 3, this range is $0.97>p_e/p_t>0.05$. This nozzle-flow configuration is known as over-expanded (see Fig. 2).
    - The shockwave is inside the nozzle if $p_{e,sub}/p_t>p_e/p_t>p_{e,es}/p_t$, where $p_{e,es}$ is the exit pressure with exit shock wave; for the example plotted in Fig. 3, $0.97>p_e/p_t>0.39$. The value of $p_{e,es}$ is found by using the normal-shockwave pressure-jump equation with $p_{e,sup}$ and $M_{e,sup}$ as upstream conditions. The sudden compression at the normal shock, and the subsequent

unfavourable pressure-gradient, makes this subsonic flow to detach from the wall, what is often termed 'grossly over-expanded flow', what yields poor nozzle performances (the typical behaviour on ground of nozzles designed for high-altitude and vacuum operation).

  - For $p_{e,es}/p_t>p_e/p_t>p_{e,sup}/p_t$, a complex structure of oblique and normal shock waves develop at the nozzle exit, with successive compressions and expansions reflecting at the free-jet boundary, gradually equalizing the pressure difference between the exhaust and the atmosphere, with the appearance of 'shock diamond' structures (Fig. 4, below). For the example plotted in Fig. 3, this range is $0.39>p_e/p_t>0.05$.

- If $p_e/p_t=p_{e,sup}/p_t$, the supersonic flow arrives at the exit with precisely the ambient-pressure value, and there are no discontinuities (other than the mixing layer around the cylindrical exhaust jet (well, slightly conical, according to the nozzle-slope at the exit).
- If $p_{e,sup}/p_t>p_e/p_t>0$, the flow is isentropic all along the nozzle length and supersonic from neck to exit section (as the case before), but the exit pressure being higher than the environment ($p_e>p_0$), a fan of Prandtl-Mayer expansion waves sets at the exit, and the flow configuration is called under-expanded (Fig. 2g).

A good example of the occurrence of the latter conditions was present in the Space Shuttle Main Engine, which leaves the pad in an over-expanded state (see the retracting exhaust jet under tests, in Fig. 4), becomes adapted (fully expanded) at high altitude, and then under expanded as the Shuttle approaches the vacuum of space.

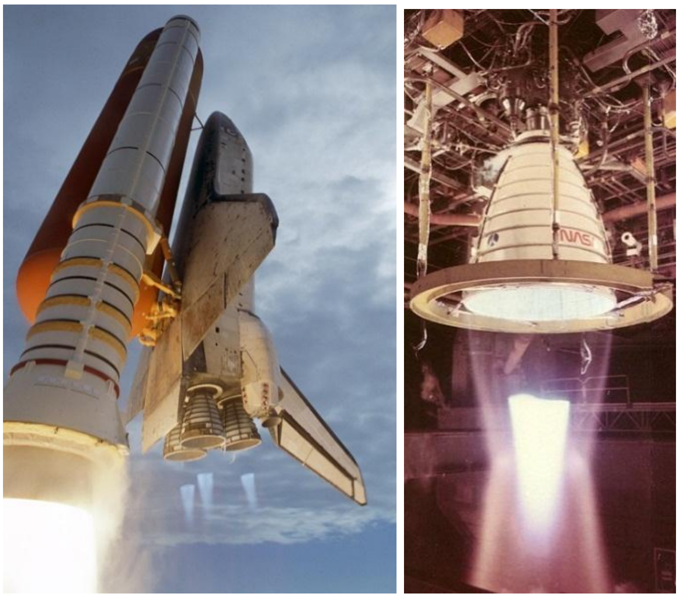


Fig. 4. Shock diamonds (blue cones) in Shuttle's main engine nozzles during STS-120 launch (NASA), and during tests.

<u>Normal shock within the nozzle</u>

A normal shock generates entropy and thus lowers total pressure (while greatly increasing static pressure). Mass flow-rate conservation relates both values: total pressure before, p_t, and after the shock, p_{te} (exit total pressure). Applying (15) to throat and exit conditions (at each side of the shock):

$$\dot{m}=\sqrt{\frac{\gamma}{R}}\frac{p_t}{\sqrt{T_t}}\left(1+\frac{\gamma-1}{2}\right)^{1-\frac{\gamma}{2}}A^*=\sqrt{\frac{\gamma}{R}}\frac{p_{te}}{\sqrt{T_t}}\left(1+\frac{\gamma-1}{2}M_e^2\right)^{1-\frac{\gamma}{2}}M_eA \quad (25)$$

now we get, instead of (14):

$$\frac{p_tA^*}{p_{te}A_e}=\frac{1}{M_e}\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_e^2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}, \quad \text{with } p_{te}=p_0\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma}{\gamma-1}} \quad (26)$$

where we have to solve for M_e for given entry and exit pressure (p_t, p_0), and throat and exit area (A^*,A_e). Once the total pressure loss computed, the actual Mach number just before the shock wave, M_s, is found from normal-shock relations:

$$\frac{p_{te}}{p_t}=\left(\frac{\frac{\gamma+1}{2}M_s^2}{1+\frac{\gamma-1}{2}M_s^2}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{\frac{\gamma+1}{2}}{\gamma M_s^2-\frac{\gamma-1}{2}}\right)^{\frac{1}{\gamma-1}} \quad (27)$$

Solving for this shock-entry Mach number, M_s, allows the spatial location of the front within the nozzle in terms of areas, from (20). In particular, the shock wave is located precisely at the exit section when the jump in Mach number across yields a downstream pressure, $p_{e,es}$ (impinging pressure is $p_{e,sup}$), equal to the environmental pressure, p_0, i.e. when:

$$\frac{p_t}{p_0}=\frac{A_e}{A^*}\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}M_2\sqrt{1+\frac{\gamma-1}{2}M_2^2}, \quad \text{with } M_2=\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2-\frac{\gamma-1}{2}}} \quad (28)$$

where M_1 and M_2 are the Mach numbers ahead and behind the normal shock wave (M_1=M_s). The specific entropy generation in the normal chock is $s_{gen}=s_2-s_1=c_p\ln(T_2/T_1)-R\ln(p_2/p_1)=-R\ln(p_{te}/p_t)$.

Exercise 5. For air expanding through a nozzle of area ratio A_{exit}/A_{throat}=4, find:

a) Mach numbers before and after a shock wave, when the shock gets stabilized inside at a section with A_s/A_{throat}=2.
b) Mach number at the exit section, and ratio of total pressures, in the case above.
c) Discharge pressure to have the normal shock at the exit.

Sol.: a) The Mach number just ahead of the shock is the supersonic solution in (20) with A_s/A^*=2, i.e. M_{ahead}=2.3. The Mach number just behind the shock is found from the normal-shock equations (28), i.e. M_{behind}=0.55.

b) The exit Mach number is the subsonic solution corresponding to M_{behind}=0.55 and area ratio A_{exit}/A_s=2; i.e. M_e=0.24. The ratio of total pressures is obtained from (26), p_{te}/p_t=0.63.

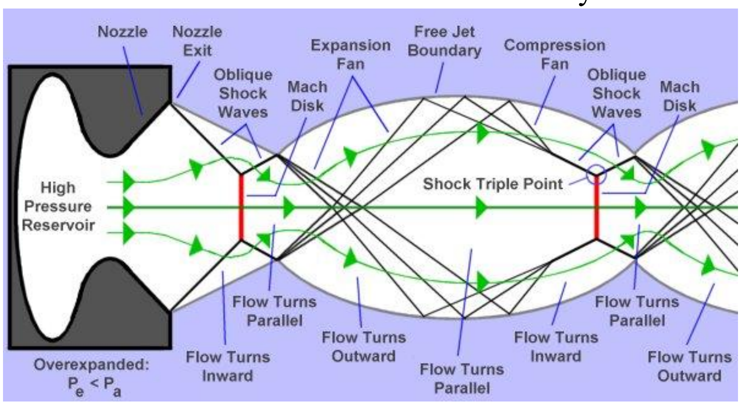
c) To have the normal shock at the exit, M_{e-}=M_1=2.9 from the supersonic solution in (20) with A_s/A^*=4, and finally M_2=M_{e+}=0.48 and p_0/p_t=0.30 from (28).

Oblique shocks at the nozzle exit

Shock diamonds (or Mach diamonds) are patterns of standing waves that appears in the supersonic exhaust plume of aerospace propulsion system (turbojet with post-combustor, solid- or liquid-fuel rocket, ramjet, or scramjet), when operated in an atmosphere with p_0>p_e (i.e. when the flow is over-expanded in a CD-nozzle.

When the exhaust flow gets across a normal shock (in red in Fig. 5a) the abrupt compression causes a sudden temperature increase, with radicals producing non-equilibrium chemiluminescence, which in the case of LH2/LOX rockets is composed of a weak emission-continuum in the blue and ultraviolet regions of the spectrum, and a 20 times stronger narrow-band emission at 310 nm, due to excitation of OH and H radicals and their recombination to H_2O. Besides, this sudden heating may cause the ignition of any residual fuel present in the exhaust, making the Mach disc and trail to glow and become visible like in Fig. 4. Behind the normal shock, the pressure is greater than that of the ambient atmosphere, so that the jet expands, trying to equalize with the external air (the expanding waves reflect off the free jet boundary and towards the centreline), what may require several expansions and compressions.

A similar process occurs in an under-expanded flow exiting from a nozzle at high altitude or under vacuum (Fig. 5b). The sequence of compression and expansion is identical to that above-described for an over-expanded nozzle, except that it begins with the creation of an expansion fan rather than oblique shock waves. This behaviour causes the flow to billow outward initially rather than spindle inward.

a) 

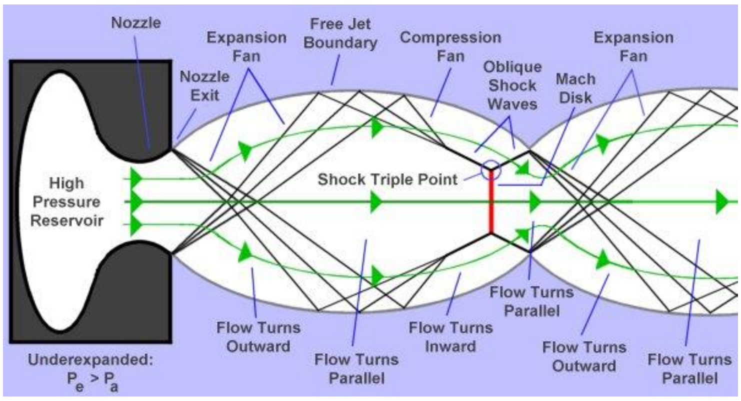
b) 

Fig. 5. a) Shock diamonds in an over-expanded flow; the first Mach disc (in red) is separated from the exit by $x/D_e = (2/3)\sqrt{p_0/p_e}$. b) Wave structures that create shock diamonds in an under-expanded flow (Aerospaceweb).

Aerospike nozzle

This is a novel design for a fix-geometry nozzle to get adapted at all altitudes. Instead of an outward flow in a bell-shape wall boundary, in the aerospike nozzle an annular flow issues radially inward along a decreasing-diameter inner wall (the spike), without external wall (after a cowl lip), see Fig. 6. The outer ambient pressure regulates the outer plume boundary so that when $p_e<p_0$ (over-expansion at low altitudes) the external pressure squeezes and makes the plume thinner, further accelerating the exhaust instead of detaching it from the walls. Since ambient pressure controls the nozzle expansion, the flow area at the end of the aerospike changes with altitude, as if it was a variable-area nozzle, and thus, a very high area ratio nozzle, which provides high vacuum performance, can also be efficiently operated at sea level. The length of an ideal spike is about 150 % of a 15° conical nozzle, but performances reduce very little if the spike length is truncated to the 20 % range, with the formation of a recirculating bubble which, if fed with a secondary jet at the base, elongates the bubble, forming an aerodynamic contour that resembles the truncated portion of the spike (this aerodynamic-shape is the reason for the "aerospike" term).

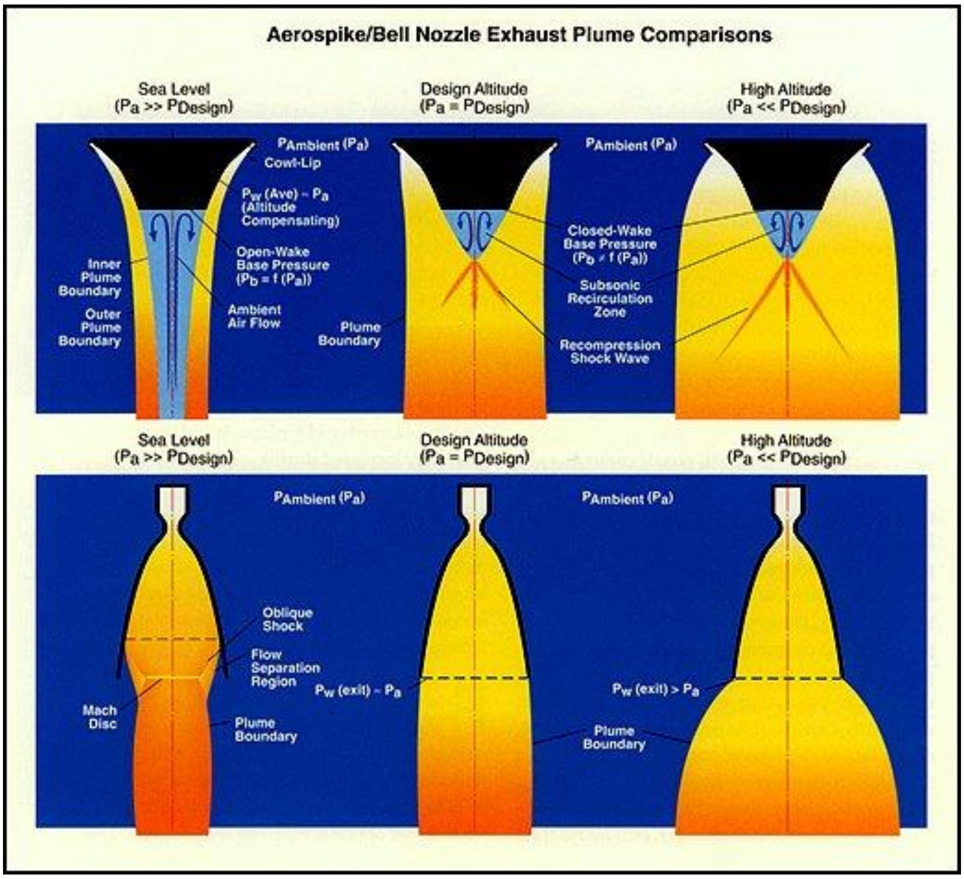


Fig. 6. Different flow configurations in aerospike nozzles, and comparison with bell nozzles.

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