RANKINE CYCLE. STEAM ENGINE

Statement

Water is pumped and feed to a boiler, starting at 100 kPa, 30 °C and ending at 1 MPa, 350 °C. The generated steam flows through a turbine with an isentropic efficiency of 0,85 and through a condenser aspirated by another pump that returns water to the initial conditions of 100 kPa and 30 °C. Find:

a) Required thermal energy, and work delivered, by unit mass flow-rate.

b) Energy and exergy efficiencies of the cycle, considering the maximum and minimum working temperatures.

c) Minimum required work (thermodynamic limit) to process water from 100 kPa and 30 °C to 1 MPa and 350 °C, in an environment at 30 °C, and comparison with consumed exergy.

d) Maximum obtainable work (thermodynamic limit) when processing water from 1 MPa and 350 °C to 100 kPa and 30 °C, in an environment at 30 °C, and comparison with work obtained.

Con una bomba y la caldera de una máquina de vapor se pasa el agua de 100 kPa y 30 °C a 1 MPa y 350 °C, entrando el vapor a una turbina de rendimiento isentrópico 0,85 y saliendo después al condensador, que está aspirado por otra bomba, la cual devuelve el agua a 100 kPa y 30 °C. Se pide:

a) Consumo térmico y producción de trabajo por unidad de gasto másico.

b) Rendimiento energético y exergético de la máquina, suponiendo que se trabaja con un foco térmico a la máxima temperatura y un sumidero térmico a la mínima temperatura.

c) Trabajo mínimo necesario (límite termodinámico) para pasar el agua de 100 kPa y 30 °C a 1 MPa y 350 °C, en presencia de una atmósfera a 30 °C, y comparación con la exergía aportada.

d) Trabajo máximo obtenible (límite termodinámico) al pasar de 1 MPa y 350 °C a 100 kPa y 30 °C, y comparación con el trabajo obtenido.

Solution.

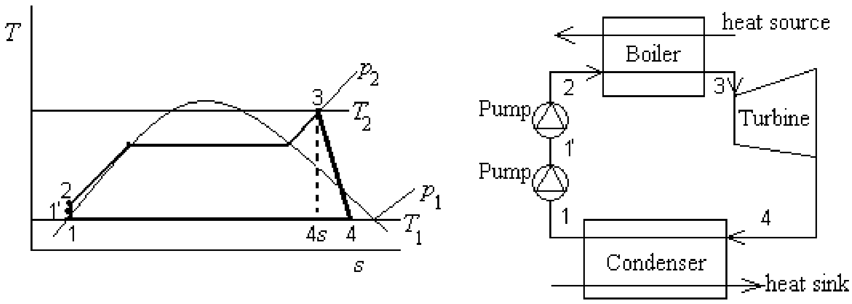

Fig. 1. Sketch of Rankine cycle and installation.

a) Required thermal energy, and work delivered, by unit mass flow-rate.

Initial state is point 1' in Fig. 1 (100 kPa, 30 ºC). The basic energy balance is $q+w=\Delta h$ for any component with one input and one output, in steady state.

Pumping water from 100 kPa to 1 MPa consumes a small work, $w=\Delta p/\rho=0.9\cdot 10^6/1000=0.9$ kJ/kg in the isentropic case, or $w=\Delta p/(\rho\eta)$ for an isentropic efficiency η; negligible in all cases against other terms to a first approximation.

Heat received in the boiler amounts to $q=h_3-h_2$, where enthalpies can be computed with the perfect substance model from the reference state of liquid at the triple point as $h_2=c_L(T_2-T_{tr})=4.2(30-0)=126$ kJ/kg and $h_3=h_{lv,tr}+c_{pv}(T_3-T_{tr})=2500+2(350-0)=3200$ kJ/kg, i.e. $h_3-h_2=3074$ kJ/kg. With the most accurate data (from NIST), $h_3-h_2=3158-126=3032$ kJ/kg.

Work delivered by the turbine is $w=h_3-h_4$, but point 4 can be either below the saturation line or above it, in the T-s (or in the h-s) diagram, and, although there is no difference to solve it graphically using the traditional Mollier diagram, solving it analytically demands a two-case approach, i.e. either assuming a single-phase output, or assuming a two-phase output (that, although more lengthy, is the most usual case), and then checking for consistency.

First, lets compute the exit pressure, that must correspond to the condensing temperature at the condenser, which must be 30 ºC because there is negligible heating in the pumps, thus $p_4=p_v$(30 ºC)=4.2 kPa, obtained by Antoine's vapour pressure fitting:

$$p_v(T)=p_u\exp\left(A-\frac{B}{C+T/T_u}\right)=(1\text{ kPa})\exp\left(16.54-\frac{3985}{-39+(30+273)}\right)=4.2\text{ kPa}$$

Second, let us start assuming that the end of the expansion in the turbine yields a wet vapour, i.e. a two-phase system. The system that solves for point 4 is:

$$\eta_T=\frac{h_3-h_4}{h_3-h_{4s}}$$

with $h_{4s}=(1-x_{4s})h_{4L}+x_{4s}h_{4V}$ and $s_3=s_{4s}=(1-x_{4s})s_{4L}+x_{4s}s_{4V}$ from where the vapour mass fraction at point 4s is deduced:

$$s_3=s_{4s}=(1-x_{4s})s_{4L}+x_4s_{4V}=(1-x_{4s})\left(s_{tr}+c_L\ln\frac{T_4}{T_{tr}}\right)+x_4\left(s_{tr}+\frac{h_{lv,tr}}{T_{tr}}+c_{p,v}\ln\frac{T_4}{T_{tr}}-R\ln\frac{p_4}{p_{tr}}\right)$$

with $s_3=s_{tr}+(h_{lv,tr}/T_{tr})+c_{pv}\ln(T_3/T_{tr})-R\ln(p_3/p_{tr})=0+(2500/273)+2\cdot\ln(623/273)-0.462\cdot\ln(1000/0.6)=$ 7.39 kJ/(kg·K), yielding $x_{4s}=0.86$, in agreement what the two-phase assumption. Enthalpy at point 4s is:

$$h_{4s}=(1-x_{4s})h_{4L}+x_{4s}h_{4V}=(1-x_{4s})\big(h_{tr}+c_L(T_4-T_{tr})\big)+x_{4s}\big(h_{tr}+h_{lv,tr}+c_{p,v}(T_4-T_{tr})\big)=$$
$$=0.14\cdot 4.2\cdot 30+0.86\cdot(2500+2\cdot 30)=2200\text{ kJ/kg}$$

and at turbine outlet $h_4=h_3+\eta_T(h_3-h_{4s})$=2340 kJ/kg, what finally yields a turbine work of $w=h_3-h_4$=3200–2340=860 kJ/kg. With the most accurate data (from NIST), h_{4s}=2207 kJ/kg, x_{4s}=0.857, h_4=2350 kJ/kg, x_4=0.915, and thence h_3-h_4=3158–2350=808 kJ/kg.

b) Energy and exergy efficiencies of the cycle, considering the maximum and minimum working temperatures.

Energy efficiency is defined and computed as:

$$\eta_e \equiv \frac{w_{net}}{q_{pos}} = \frac{w_T - w_P}{q_B} \approx \frac{w_T}{q_B} = \frac{h_3 - h_4}{h_3 - h_2} = \frac{860}{3070} = 0.28$$

whereas exergy efficiency relates to the Carnot efficiency, that depends on the maximum and minimum temperatures considered; if we take $T_{max}=T_3$=350 ºC and $T_{min}=T_1$=30 ºC, then:

$$\eta_{Carnot} \equiv 1 - \frac{T_1}{T_3} = 1 - \frac{303}{623} = 0.51 \quad \text{and} \quad \eta_x \equiv \frac{\eta_e}{\eta_{Carnot}} = \frac{0.28}{0.51} = 0.55$$

c) Minimum required work (thermodynamic limit) to process water from 100 kPa and 30 °C to 1 MPa and 350 °C, in an environment at 30 °C, and comparison with consumed exergy.

Minimum required work is equal to the exergy change, thus:

$$w_{\min 13} = \Delta\psi = (h_3 - h_1) - T_0(s_3 - s_1) = (3200 - 126) - 303(7.39 - 0.44) = 970 \text{ kJ/kg}$$

what can be compared with actual exergy input, adding the one spent in pumping plus the equivalent Carnot exergy of the heat supplied to the boiler (assumed to be at a constant temperature T_3):

$$\Delta\psi_{spent} = \Delta\psi_P + q_{23}\left(1 - \frac{T_1}{T_3}\right) = 0.9 + 3070\left(1 - \frac{303}{623}\right) = 1580 \text{ kJ/kg}$$

i.e. much more than required. And we have assumed that the hot source was just at T_3=350 ºC; in practice, it would be some 1500 ºC in the burning gases used to power the boiler, what teaches that boilers, in spite of having very good energy efficiencies (say 90% of the heating power of the fuel goes to the water), have poor exergy efficiencies.

d) Maximum obtainable work (thermodynamic limit) when processing water from 1 MPa and 350 °C to 100 kPa and 30 °C, in an environment at 30 °C, and comparison with work obtained.

Similarly, the maximum obtainable work is equal to the exergy change, thus:

$$w_{\max 31} = w_{\min 13} = 970 \text{ kJ/kg}$$

what can be compared with actual exergy delivered, adding the one produced by the turbine plus the equivalent Carnot exergy of the heat supplied by the condenser, that, assuming the environment at the same temperature T_1, adds no exergy, thus:

$$\Delta\psi_{gen} = w_T = 860 \text{ kJ/kg}$$

i.e. a little less than the maximum, what teaches that the turbine itself is quite efficient.

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