COMPARISON AMONG DIFFERENT POWER CYCLES

Statement

Consider a piston-cylinder device, holding initially 500 cm^3 of ambient air, which is to be quickly compressed to 50 cm^3. We aim at analysing the four following cyclic processes (find the energy exchanges and the energy efficiency):

a) After the compression, 1 kJ of heat is added at constant volume, and then a quick expansion follows, ending with a constant volume heat release.

b) After the compression, 1 kJ of heat is added at constant volume, and then a quick expansion follows, ending with a constant pressure heat release.

c) After the compression, 1 kJ of heat is added at constant pressure, and then a quick expansion follows, ending with a constant volume heat release.

d) After the compression, 1 kJ of heat is added at constant pressure, and then a quick expansion follows, ending with a constant pressure heat release.

Considérese un sistema cilindro-émbolo, encerrando inicialmente un volumen de 500 cm^3 de aire ambiente, el cual se va a comprimir rápidamente hasta reducir su volumen a 50 cm^3. Se quiere estudiar los cuatro procesos cíclicos siguientes (intercambios y rendimiento energéticos):

a) Tras la compresión, adición de 1 kJ de calor a volumen constante, seguido de expansión rápida hasta el volumen inicial y evacuación de calor a volumen constante.

b) Tras la compresión, adición de 1 kJ de calor a volumen constante, seguido de expansión rápida hasta la presión inicial y evacuación de calor a presión constante.

c) Tras la compresión, adición de 1 kJ de calor a presión constante, seguido de expansión rápida hasta el volumen inicial y evacuación de calor a volumen constante.

d) Tras la compresión, adición de 1 kJ de calor a presión constante, seguido de expansión rápida hasta la presión inicial y evacuación de calor a presión constante.

Solution

a) After the compression, 1 kJ of heat is added at constant volume, and then a quick expansion follows, ending with a constant volume heat release.

Let us first work out the initial compression, common to all four cycles.

From 1 to 2. It is a control-mass problem, since in air-standard cycles the mass of air is assumed to remain all the time within the cylinder. The energy balance is thus $\Delta E=W+Q$. The enclosed mass, assuming p_0=10^5 Pa and T_0=288 K, is $m=pV/(RT)$=10^5·500·10^{-6}/(287·288)=0,61 g. After the isentropic compression, the state is V_2=50 cm^3, $p_2=p_1(V_1/V_2)^{\gamma}$=10^5·(500/50)$^{1.4}$=2.5 MPa, $T_2=T_1(V_1/V_2)^{\gamma-1}$=288·(500/50)$^{0.4}$=723 K, too high for a typical fuel-air premix (stoichiometric gasoline-air mixtures ignite without the need of sparks at $T_{autoign}$=650 K), but good enogh for a compression-ignition cycle (diesel autoignites in air at T>480 K). The work the gas receives is $W_{12}=\Delta E=mc_v(T_2-T_1)$=0.61·10^{-3}· (1000-287)(723–288)=190 J, and the heat is null, Q_{12}=0. Notice that W_{12} is not the work that must be applied; ambient pressure contributes, too.

Let start now with the first cycle described, that corresponds to the well-known Otto cycle, represented in Fig. 1.

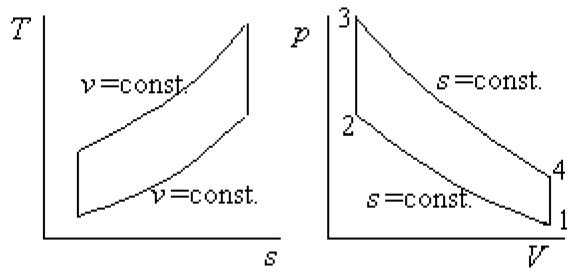

Fig. 1. Otto cycle.

From 2 to 3, Q_{23}=1 kJ is received with $V_3=V_2$ (i.e. $W_{23}=0$), and, from the energy balance $\Delta E=W+Q=mc_v(T_3\text{-}T_2)$, $T_3=T_2+Q_{23}/(mc_v)=723+1000/(0.61\cdot 10^{-3}\cdot 713)=3020$ K, too high for practical combustion engines, that only reach 2000 K, but this is a simple academic exercise. Furthermore, $p_3=p_2T_3/T_2=2.5\cdot 10^6\cdot 3020/723=11$ MPa. In fact, steel in normal reciprocating engines is kept always below 600 K by water cooling internal engine walls (air cooling in some cases); the temperature difference between the internal and external side of the cylinder walls is less than 50 K, with outer water at less than 400 K and inner gasses fluctuating cyclically in the range 1200±700 K (the fluctuations are damped in the internal wall to some ±5 K).

From 3 to 4 there is an isentropic expansion, from V_3=50 cm^3 to V_4=500 cm^3, so that $p_4=p_3(V_3/V_4)^{\gamma}=11\cdot 10^6\cdot(50/500)^{1.4}=418$ kPa, $T_4=T_3(V_3/V_4)^{\gamma-1}=3020\cdot(50/500)^{0.4}=1210$ K (too high a value, corresponding to previous temperature values). The work the gas delivers is $W_{34}=\Delta E=mc_v(T_4\text{-}T_3)=0.61\cdot 10^{-3}\cdot 713(1210\text{-}3020)=-790$ J, and the heat is null, $Q_{34}=0$. Notice again that W_{34} is not the work delivered to the shaft; ambient pressure resists, too.

From 4 to 1, heat is released at constant volume (i.e. $W_{41}=0$), and, from the energy balance $\Delta E=W+Q=mc_v(T_1\text{-}T_4)=0.61\cdot 10^{-3}\cdot 713(288\text{-}1210)=-400$ J. Notice that in real internal combustion engines there is no 4-to-1 process (the working gas is vented, and new mixture admitted). Table 1 shows the summary of energy exchanges.

Table 1. Summary of energy exchanges.

Otto	ΔE	W	Q
1-2	190	190	0
2-3	1000	0	1000
3-4	-790	-790	0
4-1	-400	0	-400
cycle	0	-600	600

The energy efficiency of a power cycle is defined as net work delivered divided by heat input, $\eta=W_{\text{net}}/Q_{\text{in}}$, that here takes the value:

$$\eta \equiv \frac{|W_{net}|}{Q_{in}} = \frac{|W_{12}+W_{23}+W_{34}+W_{41}|}{Q_{23}} = \frac{|190+0+(-790)+0|}{1000} = 0,60$$

which can be compared with the well-known efficiency of the ideal Otto cycle: $\eta=1-1/r^{\gamma-1}=1-1/10^{0,4}=0,60$, r being the compression ratio $r=V_1/V_2$ (with $r=10$ in this case).

b) After the compression, 1 kJ of heat is added at constant volume, and then a quick expansion follows, ending with a constant pressure heat release.

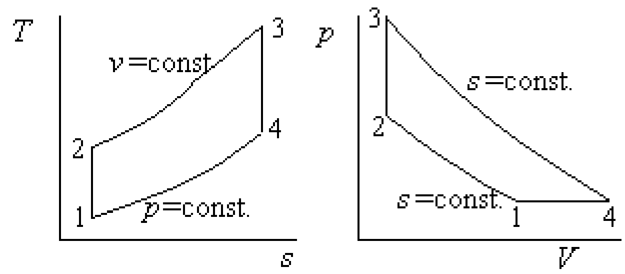

Fig. 2. New cycle.

From 1 to 2 has been worked at the very beginning: $p_1=10^5$ Pa, $T_1=288$ K, $p_2=2,5$ MPa, $T_2=723$ K, $Q_{12}=0$ and $W_{12}=190$ J.

From 2 to 3 is the same as in the Otto cycle: $p_2=2,5$ MPa, $T_2=723$ K, $p_3=11$ MPa, $T_3=3020$ K, $Q_{23}=1$ kJ and $W_{23}=0$.

From 3 to 4 there is an isentropic expansion, from $p_3=11$ MPa to $p_4=10^5$ Pa, so that $V_4=V_3(p_3/p_4)^{1/\gamma}=50\cdot10^{-6}\cdot(11/0,1)^{1/1,4}=1400\ \text{cm}^3$, $T_4=T_3(V_3/V_4)^{\gamma-1}=3020\cdot(50/1400)^{0.4}=800$ K. The work the gas delivers is $W_{34}=\Delta E=mc_v(T_4-T_3)=0.61\cdot10^{-3}\cdot713(800-3020)=-970$ J, and the heat is null, $Q_{34}=0$.

From 4 to 1, heat is released at constant pressure. The energy balance $\Delta E=W+Q$ takes the form $Q=\Delta H=mc_p(T_1-T_4)=0.61\cdot10^{-3}\cdot1000(288-800)=-310$ J, whereas $W_{41}=-p_1(V_1-V_4)=90$ J. Table 2 shows the summary of energy exchanges.

Table 2. Summary of energy exchanges.

	ΔE	W	Q
1-2	190	190	0
2-3	1000	0	1000
3-4	-970	-970	0
4-1	-220	90	-310
cycle	0	-690	690

The energy efficiency now takes the value:

$$\eta\equiv\frac{|W_{net}|}{Q_{in}}=\frac{|W_{12}+W_{23}+W_{34}+W_{41}|}{Q_{23}}=\frac{|190+0+(-970)+90|}{1000}=0,69$$

c) After the compression, 1 kJ of heat is added at constant pressure, and then a quick expansion follows, ending with a constant volume heat release.

This corresponds to the ideal Diesel cycle, depicted in Fig. 3.

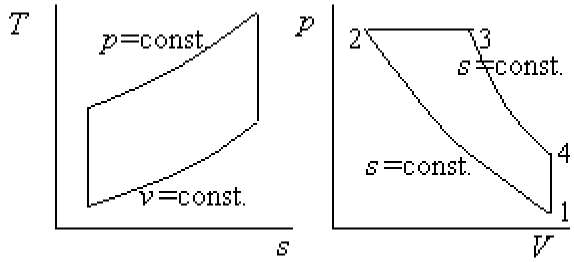

Fig. 3. Diesel cycle.

From 1 to 2 has been worked at the very beginning: p_1=10^5 Pa, T_1=288 K, p_2=2,5 MPa, T_2=723 K, Q_{12}=0 and W_{12}=190 J.

From 2 to 3, Q_{23}=1 kJ is received at constant pressure. The energy balance $\Delta E=W+Q$ takes the form $Q=\Delta H=mc_p(T_3\text{-}T_2)$, so that $T_3=T_2+Q_{23}/(mc_p)$=723+1000/(0.61·10^{-3}·1000)=2370 K and $V_3=V_2T_3/T_2$=50·10^{-6}·2370/723=164·10^{-6} m^3, with a work (output) of $W_{23}=-p_2(V_3\text{-}V_2)$=–290 J.

From 3 to 4 there is an isentropic expansion, from V_3=164 cm^3 to V_4=500 cm^3, so that $p_4=p_3(V_3/V_4)^{\gamma}$=2,5·$10^6$·$(164/500)^{1.4}$=525 kPa, $T_4=T_3(V_3/V_4)^{\gamma-1}$=2370·$(164/500)^{0.4}$=1520 K. The work the gas delivers is $W_{34}=\Delta E=mc_v(T_4\text{-}T_3)$=0.61·$10^{-3}$·713(1520-2370)=–370 J, and the heat is null, Q_{34}=0.

From 4 to 1, heat is released at constant volume (i.e. W_{41}=0), and, from the energy balance $\Delta E=W+Q=mc_v(T_1\text{-}T_4)$=0.61·$10^{-3}$·713(288-1520)=–530 J. Table 3 shows the summary of energy exchanges.

Table 3. Summary of energy exchanges.

Diesel	ΔE	W	Q
1-2	190	190	0
2-3	710	-290	1000
3-4	-370	-370	0
4-1	-530	0	-530
cycle	0	-470	470

The energy efficiency here takes the value:

$$\eta \equiv \frac{|W_{net}|}{Q_{in}} = \frac{|W_{12}+W_{23}+W_{34}+W_{41}|}{Q_{23}} = \frac{|190+(-290)+(-370)+0|}{1000} = 0,47$$

which can be compared with the well-known efficiency of the ideal Diesel cycle:

$$\eta = 1-\frac{1}{r^{\gamma-1}}\frac{r_c^{\gamma}-1}{\gamma(r_c-1)} = 1-\frac{1}{10^{0,4}}\frac{3,3^{1.4}-1}{1,4(3,3-1)} = 0,47$$

r being the compression ratio $r=V_1/V_2$ as for the Otto cycle (r=10), and r_c being the cut-off ratio, $r_c=V_3/V_2$=3,3.

d) After the compression, 1 kJ of heat is added at constant pressure, and then a quick expansion follows, ending with a constant pressure heat release.

This corresponds to the ideal Brayton cycle, depicted in Fig. 4.

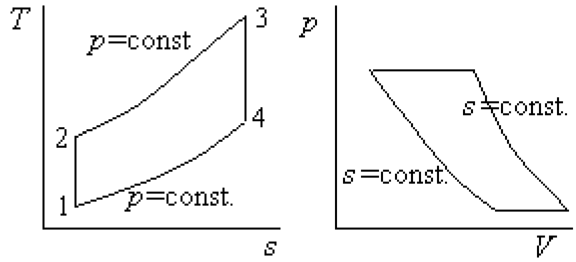

Fig. 4. Brayton cycle.

From 1 to 2 has been worked at the very beginning: $p_1=10^5$ Pa, T_1=288 K, p_2=2,5 MPa, T_2=723 K, Q_{12}=0 and W_{12}=190 J.

From 2 to 3, Q_{23}=1 kJ is received at constant pressure as in the Diesel cycle above, yielding T_3=2370 K, $V_3=164\cdot10^{-6}$ m^3, and W_{23}=–290 J.

From 3 to 4 there is an isentropic expansion, from $p_3=p_2$=2,5 MPa to $p_4=10^5$ Pa, so that $V_4=V_3(p_3/p_4)^{1/\gamma}$ $=164\cdot10^{-6}\cdot(2,5/0,1)^{1/1,4}$=1640 cm^3, $T_4=T_3(V_3/V_4)^{\gamma-1}=2370\cdot(164/1640)^{0.4}$=940 K. The work the gas delivers is $W_{34}=\Delta E=mc_v(T_4\text{-}T_3)=0.61\cdot10^{-3}\cdot713(940\text{-}2370)$=–620 J, and the heat is null, Q_{34}=0.

From 4 to 1, heat is released at constant pressure. The energy balance $\Delta E=W+Q$ takes the form $Q=\Delta H=mc_p(T_1\text{-}T_4)=0.61\cdot10^{-3}\cdot1000(288\text{-}940)$=–400 J, whereas $W_{41}=-p_1(V_1\text{-}V_4)$=114 J. Table 4 shows the summary of energy exchanges.

Table 4. Summary of energy exchanges.

Brayton	ΔE	W	Q
1-2	189	189	0
2-3	714	-286	1000
3-4	-619	-619	0
4-1	-284	114	-398
cycle	0	-602	602

The energy efficiency here takes the value:

$$\eta\equiv\frac{|W_{net}|}{Q_{in}}=\frac{|W_{12}+W_{23}+W_{34}+W_{41}|}{Q_{23}}=\frac{|189+(-286)+(-619)+114|}{1000}=0,60$$

which can be compared with the well-known efficiency of the ideal Brayton cycle:

$$\eta = 1 - \frac{1}{\pi_{12}^{(\gamma-1)/\gamma}} = 1 - \frac{1}{\left(\frac{2,5}{0,1}\right)^{\frac{1,4-1}{1,4}}} = 0,60$$

π_{12} being the pressure ratio $\pi_{12}=p_2/p_1$.

Comments

This was just a comparison between several ideal-gas cycles. Efficiencies of real engines are barely half of the ideal values.

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