VENUSIAN SATELLITE (SATÉLITE VENUSINO)

Statement

The following thermal model is to be analysed for the thermal control of a spacecraft orbiting Venus at 500 km altitude in the ecliptic plane. The probe, spin stabilised, can be approximated by a cylindrical body 70 cm in diameter and 70 cm in height. Inside it, the platforms holding the instruments (zone A in Fig. 1) is approximated as a flat cylinder of 30 cm in height, 50 kg of mass, and 1000 J/(kg.K) of thermal capacity, with 100 W of continuous electrical dissipation. The outer cylindrical wall (zone B in Fig. 1), is 1 cm thick, has 5 kg of mass with the same specific thermal capacity, an effective thermal conductivity of 5 W/m·K), and is fully covered with solar cells. Two identical base plates (C in Fig. 1) close the cylinder at both ends; the properties of these panels are to be selected to solve the thermal requirement of the instrument zone being within the operational range of 0 ºC to 50 ºC. To start with the analysis, panels C can be modelled as honeycomb structures 5 mm thick, made of aluminium ribbon 0.1 mm thick, in hexagonal cells with 5 mm from side to side, covered by thin sheets painted black. All internal faces can be modelled as black-bodies, and the assemble A+B can be considered isothermal. To do:

a) Find the external heat loads (solar, albedo and infrared) as a function of orbit position.
b) Find the thermal conductance between each plate C and the assembly A+B.
c) Find the view factors between all surfaces involved, and the radiative couplings between nodes.
d) Establish the nodal equations.
e) Find the steady temperatures at the sub-solar point.
f) Same as above but on the opposite point in the orbit.
g) Find the eclipse duration and compare it with the spacecraft relaxation time.
h) Find the temperature evolution along the orbit, in the periodic state.
i) Redesign the C panels if necessary.

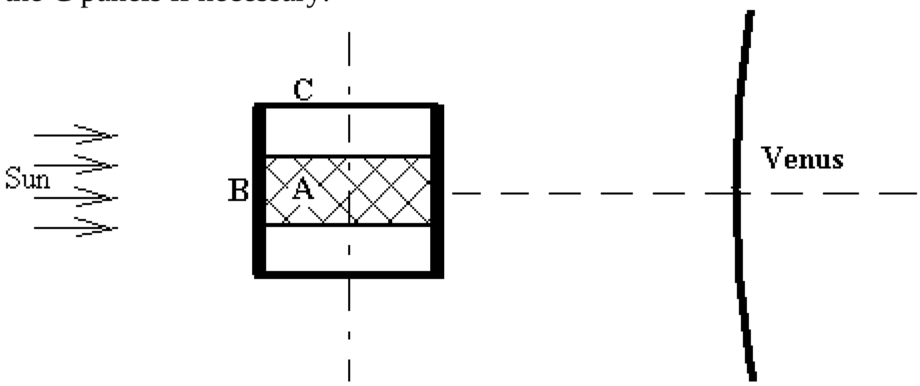

Fig. 1. Sketch of the Venusian satellite at sub-solar point in a plane perpendicular to the orbital one.

Para el estudio térmico preliminar de un pequeño satélite en órbita alrededor de Venus a 500 km de altitud y en el plano ecuatorial, se propone el siguiente modelo. Se trata de un satélite cilíndrico estabilizado por rotación, de 70 cm de diámetro y 70 cm de altura. En su interior hay una plataformas porta-instrumentos (zona A en la Fig. 1) que se aproximan a un cilindro plano de 30 cm de altura, 50 kg de masa y 1000 J/(kg·K) de capacidad térmica, con 100 W de disipación eléctrica continua. La envoltura cilíndrica exterior (zona B en la Fig. 1), tiene un grosor de 1 cm, 5 kg de masa, la misma capacidad térmica específica, una conductividad efectiva de 5 W/m·K), y está completamente cubierta con células solares. Dos placas base idénticas (C en la Fig. 1) cierran el cilindro en ambos extremos; Las propiedades de estos paneles deben seleccionarse para resolver los requisitos térmicos de la zona del instrumento dentro del rango operativo de 0 ºC a 50 ºC. Para comenzar con el análisis, los paneles C se pueden modelizar como estructuras de nido de abeja de 5 mm de espesor, hechas de hoja de aluminio de 0.1 mm de espesor, en celdas hexagonales con 5 mm de lado a lado, cubiertas por láminas delgadas pintadas de negro. Todas las caras internas pueden suponerse cuerpos negros, y el conjunto A+B puede considerarse isotérmico. Se pide:

a) Determinar las cargas térmicas (solar, de albedo e infrarroja) en función de la posición orbital.
b) Determinar la conductancia térmica entre cada nodo.
c) Determinar los factores de vista y los acoplamientos radiativos entre cada nodo.
d) Establecer las ecuaciones nodales.
e) Determinar las temperaturas que se alcanzarían con la nave en el punto subsolar, supuesto régimen estacionario.
f) Lo mismo, pero en el punto opuesto de la órbita.
g) Determinar la duración del periodo de eclipse y compararlo con el tiempo de relajación térmica de la nave.
h) Determinar la evolución de las temperaturas a lo largo de la órbita en el estado periódico.
i) Rediseñar los paneles C si fuera necesario.

Solution

We start by making a compilation of relevant data for the Venusian orbit:

Table 1. Data for the orbit around Venus.

Parameter	Symbol and value	Comments
Sun-planet distance	R_{sp}=0.72 AU	1 AU=150·10^9 m. R_{sp}=108·10^9 m mean, 109·10^9 m at aphelion and 107·10^9 m at perihelion.
Planet radius	R_p=6052 km	Nearly same size as the Earth.
Planet mass	M_p=4.87·10^{24} kg	Nearly 0.8 times that of Earth.
Solar irradiance	E=2625 W/m^2	$E_{0.72}$=E_1(1/0.72)^2=1361/0.72^2.
Albedo	ρ_p=0.76	(0.65..0.80 from different sources).
Surface temperat.	T_p=737 K	Very uniform (720..740 K).
Planet emissivity	ε_p=0.013	Emittance, $M=\varepsilon\sigma T_p^4$=210 W/m^2.
Satellite relative altitude	$h=H/R$=500/6052=0.083	Very low altitude; Venus has a thick atmosphere that reaches 250 km altitude.
Solar angle (Sun direction relative to orbit plane)	β=0	Can be taken as an equatorial orbit since Venus orbit inclination is only 3.4º (really –176.6º because it is retrograde; compare with 23.5º for the Earth).
Orbit period	T_o=5830 s (1.62 h)	$T=2\pi[(R_p+H)^3/(GM_p)]^{1/2}$, G=6.6742·10^{-11} m^3·kg^{-1}·s^{-2}.
Eclipse fraction	T_e/T_o=0.37	$T_e/T_o=(1/\pi)\arccos(((2h+h^2)^{1/2}/(1+h))=(1/\pi)\arcsin(1/(1+h))$.
Eclipse duration	T_e=2160 s (36 min)	Penumbra details are ignored.
Eclipse start angle	ϕ_{es}=1.98 rad (113º)	$\phi_{es}=\pi-\arcsin(1/(1+h))$.
Eclipse end angle	ϕ_{ee}=4.30 rad (247º)	$\phi_{ee}=2\pi-\phi_{es}$.

Notice that the angular position along the orbit, ϕ, has the origin at the sub-solar point (see the sketch in Fig. 2, which is an upside-down view; compare it with the frontal view in Fig. 1).

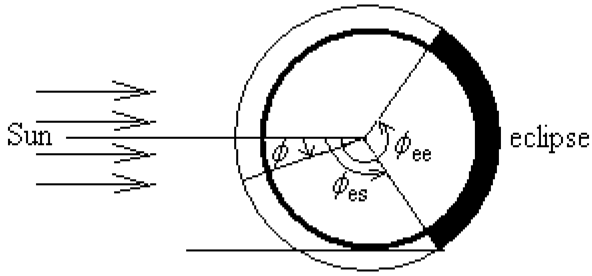

Fig. 2. Orbit view from the North Pole, to show the angular potion origin along the orbit, and the eclipse period.

We can compute and compile relevant physical parameters (geometrical and thermal) for the spacecraft.

Table 2. Data for the spacecraft.

Parameter	Symbol and value	Comments
Outer diameter	D=0.7 m	Axis pointing to ecliptic North.
Overall length	L=0.7 m	
Cylinder thickness	δ=0.01 m	m=5 kg. Mean density $\rho_{mean}=m/V$=5/0.0152=330 kg/m^3.
Base plate int. dia.	D_{int}= 0.68 m	$D_{int}=D-2\delta$. The base is assumed to fit inside.
Base plate int. area	$A_{base,int}$=0.363 m^2	$A_{base,int}=\pi(D-2\delta)^2/4=\pi\cdot(0.70-2\cdot0.01)^2/4$.
Base plate ext. area	$A_{base,int}$=0.385 m^2	$A_{base,ext}=\pi D^2/4=\pi\cdot0.70^2/4$.
Base plate thickness	δ_b=0.01 m	Mean density $\rho_{mean}=m/V$=140 kg/m^3 (from honeycomb data).
External cylind. area	A_{lat}=1.54 m^2	$A_{lat}=\pi DL=\pi\cdot0.7\cdot0.7$.
Ext. cylind. absorpt.	α=0.75	From Tables.
Ext. cylind. emissiv.	ε=0.75	From Tables.
Ext. frontal area	$A_{frontal}$=0.49 m^2.	$A_{frontal}=DL=0.7\cdot0.7$.
Ext. total area	A_{total}=2.31 m^2.	$A_{total}=\pi DL+2\pi D^2/4=\pi0.7\cdot0.7+2\pi0.7^2/4$.
Int. cyl. free area, ½	$A_{cyl,int}$=0.42 m^2	$=\pi(D-2\delta)(L-\delta_A-2\delta_b)/2=\pi\cdot0.68\cdot(0.7-0.3-2\cdot0.005)/2$.
Aluminium density.	ρ=2700 kg/m^3	
Al. thermal conduct.	k=140 W/(m·K)	It depends a lot on the alloy; this is for Al-5052.
Al. thermal capacity.	c=900 J/(kg·K)	
Al. honeycomb	ρ_{eff}=144 kg/m^3	$\rho_{eff}=\rho(8/3)\delta/s=2700\cdot(8/3)\cdot(0.0001/0.005)$.
Al. honeycomb	k_{eff}=4.2 W/(m·K)	$k_{eff}=k(3/2)\delta/s=k(3/2)\delta/s=140\cdot(3/2)\cdot(0.0001/0.005)$
Al. honeycomb	m=0.26 kg	$m=\rho_{eff}A_{face,int}\delta_2=144\cdot0.363\cdot0.005$.
Instr. plate thickness	δ_A=0.3 m	Mean density $\rho_{mean}=m/(A_A\delta_A)=50/(0.363\cdot0.3)$=460 kg/m^3.

And now we may start answering the stated questions.

a) Find the external heat loads (solar, albedo and infrared) as a function of orbit position.

Solar absorption

For an orbit in the ecliptic, a spin stabilised axisymmetric body receives a constant solar power when lit (and zero at eclipse). In our case, only the cylindrical surface gets sunshine:

$$\dot{Q}_{s,A+B}=\alpha EA_{frontal}=0.75\cdot2625\cdot0.49=965\text{ W} \quad (0 \text{ under eclipse, i.e. for } \phi \bmod 2\pi\in[\phi_{es},\phi_{ee}])$$

$$\dot{Q}_{s,C}=\alpha EA_{frontal}=0 \text{ all the time.}$$

where all the absorbed energy is assumed to be thermally dissipated within the A+B assembly. Notice that, if A and B were separately consider, a more careful analysis would be required, splitting the absorbed energy in the electrical part, α_{el}, and the thermal part, α_{th}, such that $\alpha_{el}=\eta F_{pq}$, with η being the solar cell efficiency, $\eta\equiv P_{ele,max}/(EA)$, and F_{pq} a packaging factor (ratio of active cell area to total area), and $\alpha_{th}=\alpha-\alpha_{el}$; besides, the electrical part would be a sink term in the energy balance of B (the power generator), and a source term in the energy balance of A (the power consumption equipment). However, we should check afterwards that the electrical dissipation stated, 100 W continuously, is compatible with a reasonable electricity production based on solar panel area, orbit averaged solar irradiation, and cell efficiency.

Albedo absorption

For albedo absorption, the geometrical aspects are much more complicated. In the most important and simplest case of the sub-solar point, albedo absorption is proportional to reflected power, $\rho_p E_s$ (with Venus albedo ρ_p=0.76 found in Tables), an effective area defined in terms of the view factor to the planet, $F_{b,p}$, another view factor related to the planet phase as seen from the body, F_a, and body absorptance, α, i.e.:

$$\dot{Q}_a=\alpha F_a A_p F_{p,b}\rho_p E=\alpha F_a A_b F_{b,p}\rho_p E$$

For the base plates C, there is an analytical expression for the view factor from a small patch to a large sphere:

$$F_{C,p}=\frac{1}{\pi}\left(\arctan\frac{1}{\sqrt{2h+h^2}}-\frac{\sqrt{2h+h^2}}{(1+h)^2}\right)\overset{h=0.083}{=}0.26$$

The albedo factor, F_a, accounting for the partial sunlit of the planet out of the subsolar point, we may just use a cosine law, $F_a=\cos\phi$ (when positive, i.e. $F_a=0$ in the range ϕ mod $2\pi\in[\pi/2,3\pi/2]$, having neglected albedo contributions close to eclipse transitions). If we assume C to be a blackbody, $\alpha_C=1$, albedo input becomes:

$$\dot{Q}_{a,C}=\alpha F_a A_C F_{C,p}\rho_p E=1\cdot\cos\phi\cdot 0.385\cdot 0.26\cdot 0.76\cdot 2625=200\cdot\cos\phi\ \text{[W]}$$

with $\dot{Q}_{a,C}=0$ in the range ϕ mod $2\pi\in[\pi/2,3\pi/2]$ as said. Notice that we assign the whole base area to panel C (A_C=0.385 m^2) without discounting the rim area that we assumed belonging to the cylinder.

For the cylindrical surface, there is no analytical expression for its view factor towards a large sphere, although in the case of an untilted cylinder, it can be reduced to a quadrature with complete elliptic integrals of the second kind E(x):

$$F_{B,p}=\frac{4}{\pi^2}\int_{\beta=0}^{\arcsin\left(\frac{1}{1+h}\right)}\sin(\beta)\mathrm{E}(\sin(\beta))\,\mathrm{d}\beta$$

with a value of $F_{B,p}$=0.34 for h=0.083.

Instead of the integration, a good approximation may be to use the view factor from a small sphere to a large sphere, since our cylinder has a slenderness H/R=1, obtaining:

$$F_{C+B+C,p} \approx \frac{1}{2}\left(1-\frac{\sqrt{2h+h^2}}{1+h}\right) \overset{h=0.083}{=} 0.31$$

Notice that the global view factor for our satellite, found by composition, $F_{C+B+C,p}=(2A_CF_{C,p}+A_BF_{B,p})/(2A_C+A_B)$=(2·0.385·0.26+1.54·0.34)/(2·0.385+1.54)=0.31, is practically equal to the simplest spherical model used above.

Another, more general, approximation may be to assimilate the cylinder to a square prism of the same lateral area ($\pi D=4L'$, i.e. of side $L'=0.7\pi/4$=0,55 m), and use the available view factors for the frontal face, $F_{12}=1/(1+h)^2=1/(1+0.083)^2$=0.85, and combine it with the level faces computed above, to yield:

$$A_BF_{B,p} = A_{B,frontal}F_{B,frontal,p} + 2A_{B,lateral}F_{B,lateral,p} \overset{h=0.083}{=} (0.55\cdot 0.7)\cdot 0.85 + 2\cdot(0.55\cdot 0.7)\cdot 0.26 =$$

$$= 0.53\text{ m}^2 \quad \Rightarrow \quad F_{B,p} = \frac{A_{A+B}F_{A+B,p}}{A_{lat}} = \frac{0.53}{1.54} = 0.34$$

Albedo absorption by the cylindrical body, covered with solar cells with α=0.75 (we take the beginning-of-life value from Thermo-optical data), would be then:

$$\dot{Q}_{a,B} = \alpha F_a A_B F_{B,p} \rho_p E = 0.75\cdot\cos\phi\cdot 1.54\cdot 0.34\cdot 0.76\cdot 2625 = 775\cdot\cos\phi \text{ [W]}$$

with $\dot{Q}_{a,B}=0$ in the range ϕ mod $2\pi \in [\pi/2, 3\pi/2]$ as above. Notice that the whole albedo load at the subsolar point (F_a=1), 775+2·200=1175 W, is greater than the direct solar load, 965 W, because of the large albedo of Venus. Notice also that we have assumed again that all the energy absorbed goes to heating in the A+B assembly, since electricity production and consumption occur inside the system.

Planet IR absorption

This problem is much easier than the albedo one, because now the input is constant along the orbit. The view factors from object to planet have already been calculated for the albedo input, and there is no planet-phase effect here, i.e.:

$$\dot{Q}_p = \alpha_{FIR} A_p F_{p,b} M_p = \varepsilon A_b F_{b,p} \varepsilon_p \sigma T_p^4 \begin{cases} \dot{Q}_{p,C} = 1\cdot 0.385\cdot 0.26\cdot 0.013\cdot 5.67\cdot 10^{-8}\cdot 735^4 = 22\text{ W} \\ \dot{Q}_{p,B} = 0.75\cdot 1.54\cdot 0.34\cdot 0.013\cdot 5.67\cdot 10^{-8}\cdot 735^4 = 85\text{ W} \end{cases}$$

An overall view of the three heat loads is presented in Table 3, and the variation along the orbit position is presented in Fig. 3 with equal scales to see the relative importance.

Table 3. Summary of maximum environmental heat loads on the spacecraft.

	Solar	Albedo	Planet IR	Total

On each panel C	0	200 W	22 W	222 W
On cylindrical surface B	965	775 W	85 W	1825 W
On whole spacecraft (2·C+B)	965	1175 W	129 W	2269 W

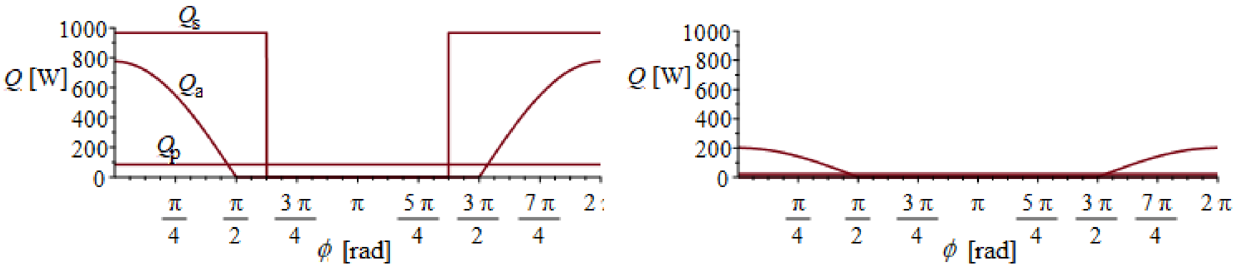

Fig. 3. Heating power received by direst solar radiation, Q_s, by albedo, Q_a, and by planet IR radiation, Q_p,), as a function of angular potion in the orbit, ϕ, with ϕ=0 at the subsolar point. a) At the cylindrical surface (B). b) At each of the two bases (C).

b) Find the thermal conductance between each plate C and the assembly A+B.

The goal is now to compute G_{BC} in $\dot{Q}_{cond} = G_{BC}(T_B - T_C)$, which, in the quasi-one-dimensional model is:

$$\dot{Q}_{cond} = k_B A_B \frac{T_B - T_{joint}}{L_B} = k_2 A_2 \frac{T_{joint} - T_C}{L_C} = G_{BC}(T_B - T_C) \quad \Rightarrow \quad G_{BC} = \frac{1}{\frac{L_B}{k_B A_B} + \frac{L_C}{k_C A_C}}$$

where A stands for the heat-flow cross-section area, and L for the characteristic length along the heat-flow path. In our case $A_B=\pi D\delta_b=\pi\cdot0.7\cdot0.01=0.022$ m^2 and $A_C=\pi D\delta_c=\pi\cdot0.7\cdot0.01=0.011$ m^2 at the interface; the path along B is just $L_B=L/2=0.35$ m, but for the path along C one has to consider the radial spreading, and set L_C=area/perimeter=$(\pi D^2/4)/(\pi D)=D/4=0.175$ m. The effective conductivity along the honeycomb C can be computed as $k_{eff}=k\delta/s$ (apart from a numeric factor of order one, depending on the orientation of the cells; see details in Honeycom panels), with k being the conductivity of the foil material making the honeycomb (in our case an aluminium alloy with say k=140 W/(m·K)), δ the foil thickness (0.1 mm) and s the side-to-side distance in a cell (5 mm). Substituting values we have:

$$k_C=k_{eff}=k\delta/s=140\cdot(0.0001/0.005)=2.8 \text{ W/(m·K)}$$

and hence:

$$G_{BC} = \frac{1}{\frac{L_B}{k_B A_B} + \frac{L_C}{k_C A_C}} = \frac{1}{\frac{0.35}{5\cdot0.022} + \frac{0.175}{2.8\cdot0.011}} = 0.113 \ \frac{\text{W}}{\text{K}}$$

c) Find the view factors between all surfaces involved, and the radiative couplings between nodes.

Although it is only the view factors between isothermal surfaces what is needed, and in our case this is a trivial question because panels C only see isothermal half-spaces (A+B, and deep space), we develop here a three-body model: A, B, and C, which, because of the mid-plane symmetry reduce to the half-body sketch shown in Fig. 4.

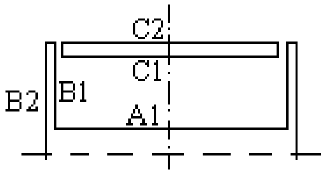

Fig. 4. Sketch of the half-satellite geometry for view factor calculations.

The view factor between the two concentric circular faces C1 and A1 is:

$$F_{12}=\frac{x-y}{2},\ y=\sqrt{x^2-4r_2^2/r_1^2},\ x=1+1/r_1^2+r_2^2/r_1^2,\ r_1=R_1/H,\ r_2=R_2/H$$

In our case, the separation is $H=(L-2\delta_C-\delta_A)/2=(0.7-2\cdot0.005-0.3)/2=0.195$ m, and the radii $R_1=R_2=R=(D-2\delta)/2=(0.7-2\cdot0.01)/2=0.34$. Thence, $r_1=r_2=R/H=0.34/0.195=1.74$, $x=1+1/1.74^2+1=2.33$, $y=(2.33^2-4)^{1/2}=1.20$, and finally $F_{A1,C1}=F_{C1,A1}=(2.33-1.20)/2=0.57$.

The view factor from one of the discs to the cylindrical surface, $F_{A1,B1}=F_{C1,B1}$, can be obtained by the closeness relation, $F_{A1,B1}+F_{A1,C1}=1$, thus $F_{A1,B1}=1-F_{A1,C1}=1-0.57=0.43$. By the reciprocity relation, $A_{A1}F_{A1,B1}=A_{B1}F_{B1,A1}$, and thus, with $A_{A1}=A_{face,int}=0.363$ m^2 and $A_{B1}=\pi(D-2\delta)H=\pi(0.7-2\cdot0.01)\cdot0.195=0.42$ m^2, $F_{B1,A1}=A_{A1}F_{A1,B1}/A_{B1}=0.365\cdot0.43/0.42=0.37$.

The view factors from B1 must verify the closeness relation, $F_{B1,A1}+F_{B1,B1}+F_{B1,C1}=1$, so that $F_{B1,B1}=1-F_{B1,A1}-F_{B1,C1}=1-0.37-0.37=0.26$, what completes the view factor matrix:

$$\begin{pmatrix} F_{A1,A1} & F_{A1,B1} & F_{A1,C1} \\ F_{B1,A1} & F_{B1,B1} & F_{B1,C1} \\ F_{C1,A1} & F_{C1,B1} & F_{C1,C1} \end{pmatrix}=\begin{pmatrix} 0 & 0.43 & 0.57 \\ 0.37 & 0.26 & 0.37 \\ 0.57 & 0.43 & 0 \end{pmatrix}$$

Notice that if the view factor from node C to node A+B was inquired, the answer is simple: half of the emission from C goes to the background and the other half goes to node A+B. However, the area ratio and the reciprocity rule dictate that $F_{A+B,C}=2A_{C1}F_{C1,A+B}/(A_{A1}+A_{B1}+A_{B2})=$ $2\cdot0.363\cdot0.5/(0.363+0.417+0.770)=0.234$, i.e. only 23.4 % of what emits A+B impinges on C. Another $F_{A+B,\infty}=A_{B2}/(A_{A1}+A_{B1}+A_{B2})=49.7$ % goes out to space, and the remaining 26.9 % goes from A+B to A+B. Notice that in some cases we have neglected the rim area of the cylinder (of width $\delta_B=1$ cm) against the other areas (A_{A1}, A_{B1}, andA_{B2}).

Radiative couplings

We want to find the heat transfer by radiation from an isothermal black-body surface A_1 at T_1 to another isothermal black-body surface A_2 at T_2, which, in view of the fourth-power Stefan-Boltzmann's emission law, we may set as $\dot{Q}_{12}=\sigma R_{12}\left(T_1^4-T_2^4\right)$, where R_{12} is a so-defined radiative coupling. This radiative heat transfer is the net radiation flow $\dot{Q}_{12}=\sigma R_{12}\left(T_1^4-T_2^4\right)$. In our case of black-body surfaces, radiative coupling only depends on view factors, $R_{12}=A_1F_{12}=A_2F_{21}$. The computation is performed in the following point.

d) Establish the nodal equations.

The thermal-energy balance of each element (nodal equations) is:

$$m_i c_i \frac{\mathrm{d}T_i}{\mathrm{d}t} = \dot{W}_{i,dis} + \dot{Q}_i = \dot{W}_{i,dis} + \dot{Q}_{i,con} + \dot{Q}_{i,rad} = \dot{W}_{i,dis} + \sum_j \dot{Q}_{i,j,con} + \sum_j \dot{Q}_{i,j,rad} + \dot{Q}_{i,s} + \dot{Q}_{i,a} + \dot{Q}_{i,p} - \dot{Q}_{i,\infty}$$

where the radiation loads from the Sun, albedo, and planet emission, have been separated from the rest of the radiative couplings (care must be paid to avoid counting radiative couplings twice, or none at all).

In particular, the nodal equation for one of the panels C (black body) is:

$$m_C c_C \frac{\mathrm{d}T_C}{\mathrm{d}t} = \dot{W}_{C,dis} + \dot{Q}_{C,B,con} + \dot{Q}_{C,B,rad} + \dot{Q}_{C,A,rad} + \dot{Q}_{C,s} + \dot{Q}_{C,a} + \dot{Q}_{C,p} - \dot{Q}_{C,\infty} =$$
$$= 0 + G_{\mathrm{B,C}}\left(T_{A+B} - T_C\right) + A_{\mathrm{C}} F_{\mathrm{C,A+B}} \sigma \left(T_{A+B}^4 - T_C^4\right) + 0 + \dot{Q}_{C,a} + \dot{Q}_{C,p} - A_{\mathrm{C}} \sigma T_C^4$$

where the last term accounts for the energy emitted by C from its external surface, in all directions, including that towards the planet, since the planetary term only accounts for the gross input, not net input. Collecting all the values previously calculated ($\dot{Q}_{C,a}$=200 W with a cosine factor variation along the lit orbit, $\dot{Q}_{C,p}$=22 W, $F_{\mathrm{A+B,C}}A_{\mathrm{A+B}}=F_{\mathrm{C,A+B}}A_{\mathrm{C}}=1\cdot0.363=0.363$ m^2, $G_{\mathrm{B,C}}=0.083$ W/K), we have (in SI units):

$$0.24\cdot 900 \frac{\mathrm{d}T_C}{\mathrm{d}t} = 0 + 0.11\cdot\left(T_{A+B} - T_C\right) + 2.06\cdot 10^{-8}\left(T_{A+B}^4 - T_C^4\right) + 0 + 200\cdot\cos\phi\cdot\left(\cos\phi > 0\right) + 22 - 2.06\cdot 10^{-8} T_C^4$$

On the other hand, the energy balance for the assembly A+B is:

$$m_{\mathrm{A+B}} c_{\mathrm{A+B}} \frac{\mathrm{d}T_{\mathrm{A+B}}}{\mathrm{d}t} = \dot{W}_{A+B,dis} - 2\dot{Q}_{C,A+B,con} - 2\dot{Q}_{C,A+B,rad} + \dot{Q}_{\mathrm{A+B},s} + \dot{Q}_{\mathrm{A+B},a} + \dot{Q}_{\mathrm{A+B},p} - \dot{Q}_{\mathrm{A+B},\infty}$$

where the radiative and conductive couplings with the two end plates has been put in terms of the corresponding previously-computed terms for one end-face. Notice again we should not account for the electrical energy dissipation in A because we have taking the gross solar absorption in the solar cells, and the 100 W are internal to the A+B system. Substituting values we have (in SI units):

$$\left(50+5\right)\cdot 1000 \cdot \frac{\mathrm{d}T_{\mathrm{A+B}}}{\mathrm{d}t} = 0 - 2\cdot 0.11\cdot\left(T_{\mathrm{A+B}} - T_C\right) - 2\cdot 2.06\cdot 10^{-8}\cdot\left(T_{\mathrm{A+B}}^4 - T_C^4\right) +$$
$$+ 965\cdot\left(\left|\phi\right| < 1.98\right) + 783\cdot\cos\phi\cdot\left(\cos\phi > 0\right) + 85 - 8.73\cdot 10^{-8}\cdot T_{\mathrm{A+B}}^4$$

Mind that the improper notation $|\phi|<\phi_e$=1.98 rad has been quoted for simplicity in writing, but the proper ϕ-dependence shown in Fig. 3 has been used in the computations. The relation between time and angle, both with origin at the sub-solar point, is $t/T_o=\phi/(2\pi)$, with the orbit period T_o=5830 s.

e) Find the steady temperatures at the sub-solar point.

We look for the solution to the two nodal equations in static conditions at ϕ=0, i.e. namely:

$$0=0+0.11\cdot\left(T_{A+B}-T_C\right)+2.18\cdot10^{-8}\left(T_{A+B}^4-T_C^4\right)+0+200+22-2.06\cdot10^{-8}T_C^4$$
$$0=0-2\cdot0.11\cdot\left(T_{A+B}-T_C\right)-2\cdot2.18\cdot10^{-8}\left(T_{A+B}^4-T_C^4\right)+965+775+85-6.55\cdot10^{-8}T_{\mathrm{A+B}}^4$$

The system must be solved by iterations, and yields $T_{\mathrm{A+B}}$=392 K (118 ºC) and T_{C}=360 K (87 ºC), although it can be solved explicitly in T^4 if the conductive coupling is neglected, what would yield almost the same results. Notice that $T_{\mathrm{A+B}}$=118 ºC is well over the warm-operation limit of 50 ºC initially established.

If the whole satellite was supposed isothermal (a one-node model), the energy balance would have been:

$$mc\frac{\mathrm{d}T}{\mathrm{d}t}=\dot{W}_{dis}+\dot{Q}_s+\dot{Q}_a+\dot{Q}_p-\dot{Q}_\infty \quad\rightarrow\quad 0=0+965+1175+129+10.9\cdot10^{-8}T^4$$

where the loads on A+B and on the two plates C, have been added, and the emission from the whole satellite area A_{total}=1.54+2·0.385=2.31 m^2, with a weighted average emissivity (assumed to be $\varepsilon_{\mathrm{mean}}$=(0.75·1.54+2·0.385)/2.31=0.83) has been accounted for, yielding a value T=380 K (107 ºC).

f) Same as above but on the opposite point in the orbit.

Taking out solar and albedo contributions, the previous system becomes:

$$0=0+0.11\cdot\left(T_{A+B}-T_C\right)+2.06\cdot10^{-8}\left(T_{A+B}^4-T_C^4\right)+0+0+22-2.06\cdot10^{-8}T_C^4$$
$$0=0-2\cdot0.11\cdot\left(T_{A+B}-T_C\right)-2\cdot2.06\cdot10^{-8}\left(T_{A+B}^4-T_C^4\right)+0+0+85-8.73\cdot10^{-8}T_{\mathrm{A+B}}^4$$

with the result $T_{\mathrm{A+B}}$=187 K (–86 ºC) and T_{C}=183 K (–90 ºC). Notice that $T_{\mathrm{A+B}}$=–86 ºC is well below the cold-operation limit of 0 ºC initially established. Again, with the one-node model:

$$0=0+0+0+129-10.7\cdot10^{-8}T^4 \quad\Rightarrow\quad T=186\text{ K}$$

We can see that Venusian orbit is hot on the sunlit side (and that albedo contributes more than the direct sunshine), but very cold in the shadow side, in spite of the very high temperature of Venus' surface, because the great greenhouse effect greatly decreases planet emittance (albedo radiates $\rho_{\mathrm{p}}E$=2014 W/m^2, whereas planet infrared emission is just $\varepsilon\sigma T^4$=0.013·5.67·10^{-8}·735^4=215 W/m^2.

g) Find the eclipse duration and compare it with the spacecraft relaxation time.

The eclipse duration was calculated at the beginning T_{e}=2170 s (36 minutes), from the eclipse fraction, T_e/T_o=(1/π)arccos(((2h+h^2)$^{1/2}$/(1+h))=(1/π)arcsin(1/(1+h))=0.37, and the orbit period, T_o=2π[(R_{p}+H)^3/(GM_{p})]$^{1/2}$=2π[(6052+500)^3/(6.7·10^{-11}·4.87·10^{24})]$^{1/2}$=5830 s (1.6 h). These values are very similar to low Earth orbit values (about T_o=1.5 h and T_e=34 min for LEO).

The thermal relaxation time of a body is the time it takes to heat up or cool down when the boundary conditions are changed (or the initial conditions were not that of equilibrium). As heat losses to the environment (and most of the times heat gains from heaters or the environment) are dependent on a temperature difference, the relaxation time would theoretically take an infinite time, and the only interest is on a finite time 'representative' of the relaxation process, usually the time it takes for the initial temperature jump (difference between the initial and final states) to reduce to a half, or the time it would take to bridge the temperature gap with the initial rate; for a simple exponential response, $\Delta T=\Delta T_0\exp(-t/t_r)$, the initial jump shortens to a half after $t_r\ln 2=0.69\cdot t_r$, whereas at the initial rate $\mathrm{d}\Delta T/\mathrm{d}t|_0=-\Delta T_0/t_r$, the jump would last t_r, corresponding with the real rate law to a jump reduction from ΔT_0 to $\Delta T_0/e=0.37\cdot\Delta T_0$; the difference is not important for an order of magnitude analysis.

In our case, the interest on relaxation time is to elucidate if there is time enough to cool down from sub-solar hot-conditions to the cold conditions at the opposite point, i.e. if the thermal inertia of the satellite allows to cool down from the average 380 K found in §*e* to the average 181 K of §*f* in about T_e=2170 s (36 min). To avoid the need of numerical computations, a first approximation may be to analyse the cooling of the satellite from some initial conditions (e.g. the sub-solar calculated T=380 K) in the deep-space environment of negligible temperature (really T_∞=2.7 K), what can be solved either by order-of-magnitude analysis, or by integration:

$$mc\frac{\mathrm{d}T}{\mathrm{d}t}=-A\varepsilon\sigma T^4\begin{cases}\xrightarrow{\text{OMA}} mc\dfrac{T_0/2}{t_r}\approx A\varepsilon\sigma T_0^4 & \Rightarrow\quad t_r=\dfrac{mc}{2A\varepsilon\sigma T_0^3}\\ \xrightarrow{\text{ODE}} mc\dfrac{\mathrm{d}T}{T^4}=-A\varepsilon\sigma\mathrm{d}t & \rightarrow\quad \dfrac{1}{T^3(t)}-\dfrac{1}{T_0^3}=\dfrac{3A\varepsilon\sigma}{mc}t\quad\Rightarrow\quad t_r=\dfrac{7mc}{3A\varepsilon\sigma T_0^3}\end{cases}$$

Substituting in the last result m=55 kg, c=1000 J/(kg/K), A=2.31 m^2, ε=0.83 (the average), and T_0=380 K, one gets a relaxation time of t_r=4600 s (1 h and a quarter) according to the first guess, and of t_r=21 500 s (6 h) according to the second guess, more realistic because the cooling rate decreases rapidly with decreasing temperature (it is proportional to T^4). A graph of the temperature evolution, Fig. 5, may clarify the point.

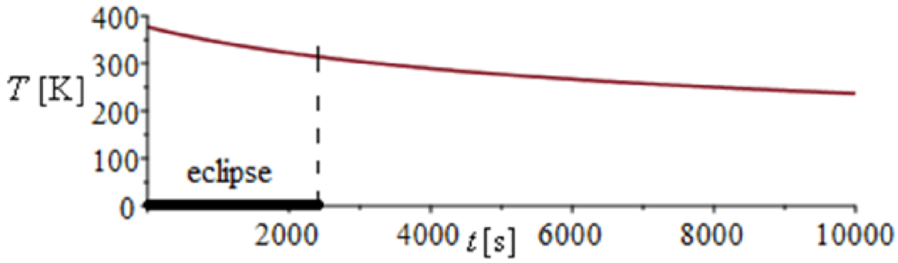

Fig. 5. Cooling down of our spacecraft, assumed isothermal (one node), if all the heat loads were absent, starting from T_{ini}=380 K at t=0, into a 2.7 K environment (Venusian eclipse only lasts 36 min, i.e. 2170 s).

h) <u>Find the temperature evolution along the orbit, in the periodic state.</u>
It is just a matter of numerically solving the system of two ordinary differential equations above obtained:

$$0.24\cdot 900\frac{\mathrm{d}T_C}{\mathrm{d}t}=0+0.18\cdot\left(T_{A+B}-T_C\right)+2.06\cdot 10^{-8}\left(T_{A+B}^4-T_C^4\right)+0+200\cdot\cos\phi\cdot\left(\cos\phi>0\right)+$$
$$+16-2.06\cdot 10^{-8}T_C^4$$

$$55\cdot 1000\cdot\frac{dT_{A+B}}{dt}=0-2\cdot 0.18\cdot\left(T_{A+B}-T_C\right)-2\cdot 2.06\cdot 10^{-8}\cdot\left(T_{A+B}^4-T_C^4\right)+$$
$$+965\cdot\left(|\phi|<1.98\right)+783\cdot\cos\phi\cdot\left(\cos\phi>0\right)+85-8.73\cdot 10^{-8}\cdot T_{A+B}^4$$

with the change $t/T_o=\phi/(2\pi)$ (T_o=5830 s), and appropriate initial conditions. If may start with the steady values obtained before, but they do not correspond to the dynamic, periodic, sub-solar-point values, thus, we start with a coarse guess, $T_{A+B}=T_C$=300 K (27 ºC), and integrate the two coupled differential equations using a simple Euler method, obtaining the results present in Fig. 6a, where we see that a periodic solution (Fig. 6b) is reached after a couple of orbit periods.

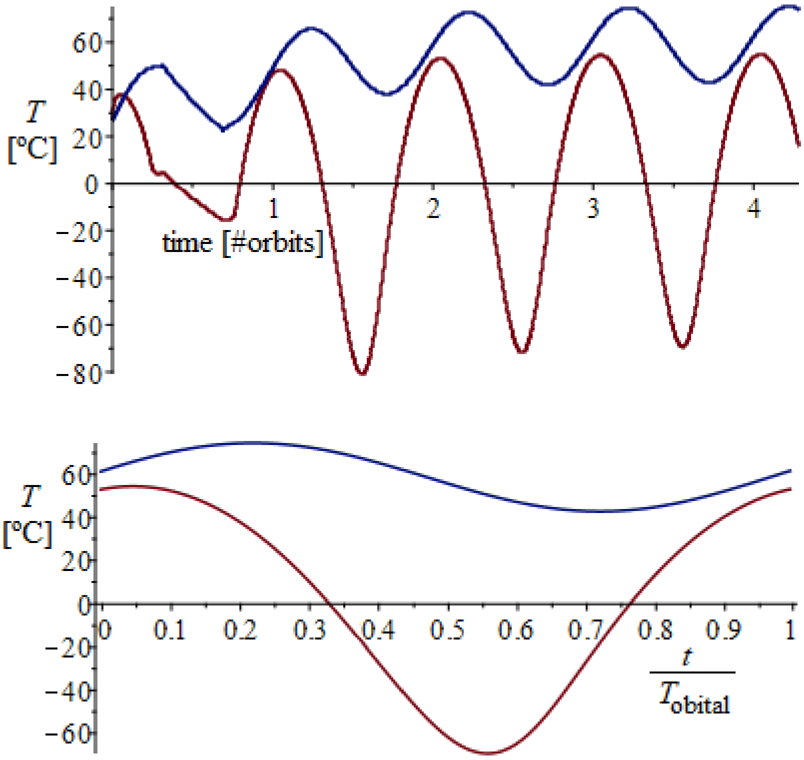

Fig. 6. a) Temperature evolution along the orbit, for the two nodes (A+B, in blue, and C, in red), starting with the whole body at 300 K, showing the transitory orbits until the periodic solution. b) Periodic solution.

The results show that the intended operational temperature range for the electronics platform, 0 ºC < T_{A+B} < 50 ºC, is exceeded during most of the orbit period (its mean value is $T_{A+B,mean}$=59 ºC). Notice that dynamical simulation predicts maximum temperatures of $T_{A+B,max}$=75 ºC and $T_{C,max}$=55 ºC, taking place after the sub-solar point, whereas the static analysis at the subsolar point yielded $T_{A+B,max}$=118 ºC and $T_{C,max}$=87 ºC. Similarly, dynamical simulation predicts minimum temperatures of $T_{A+B,min}$=43 ºC and $T_{C,min}$=−69 ºC, taking place near the end of the eclipse, whereas the minimum static values were $T_{A+B,min}$=−86 ºC and $T_{C,min}$=−90 ºC, clearly showing the effect of the much larger thermal inertia of A+B.

i) Redesign the C panels if necessary.

If the thermal conductance between the main body A+B and the polar plates C, which act as radiators (emit much more energy than what they get), could be increased, then both temperature-profiles would approach each other (see Fig. 6), and the equipment platform might be kept within limits.

To that aim, and in order not to increase the mass substantially, the best approach may be the implementation of highly conducting strips joining A+B with C.

Back to Spacecraft thermal control
Back to Heat and mass transfer
Back to Thermodynamics