TWO NON-ADJACENT PERPENDICULAR PLATES (DOS PLACAS PERPENDICULARES SEPARADAS)

Statement

Consider the radiative coupling between two equal-size plates (1 and 2 in Fig. 1) of $0.25 \cdot 0.50$ m^2. The plates are separated 0.25 m from the common line of their corresponding planes. Plate 1 has α=0,6 y ε=0,8 at both faces, and gets 800 W/m^2 normal to its external face (from a collimated source), whereas plate 2 has ε=0,5 at its upper face, and ε=0,8 at the lower one. Find:

a) All the view factors involved.
b) The required power to be applied to plate 2 in order to keep it at 20 ºC in an environment that may either be at 2.7 K or at 15 ºC.
c) Temperature of plate 1 in each case.

Enunciado

Considérese la radiación térmica entre las placas 1 y 2 de la Fig. 1. Ambas placas son de $0,25 \cdot 0,50$ m^2, y están desplazadas 0,25 m respecto a la línea de intersección de los planos que las contienen. La placa 1 tiene α=0,6 y ε=0,8 por ambas caras, y por su cara exterior absorbe 800 W/m^2 de radiación perpendicular (proveniente de una fuente colimada), mientras que la placa 2 tiene ε=0,5 por la cara superior y ε=0,8 por la inferior. Se pide:

a) Factores geométricos.
b) Potencia necesaria para mantener la placa 2 a 20 ºC en un entorno a 2,7 K, y en un entorno a 15 ºC.
c) Temperaturas de la placa 1 en cada caso.

Solution

Note. A variation of this problem is solved aside.

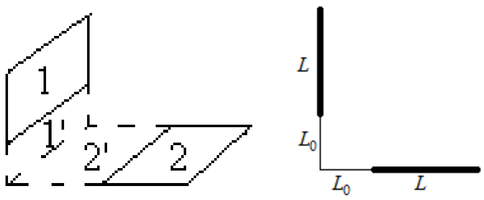
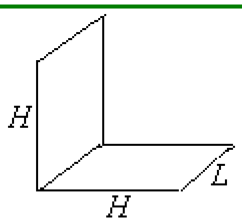


Fig. 1. Geometry.

a) All the view factors involved.

We may find tabulated the view factor between adjacent equal plates, i.e. $F_{1'\to2'}$ and $F_{1+1'\to2+2'}$:

Table 1. View factor between adjacent equal rectangles at 90º, of height H and width L, with $h \equiv H/L$.



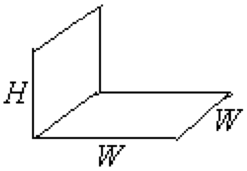
$$F_{1\to2} = \frac{1}{\pi}\left[2\arctan\left(\frac{1}{h}\right) - \sqrt{2}\arctan\left(\frac{1}{\sqrt{2}h}\right) + \frac{1}{4h}\ln\left(\frac{h_1 h_2}{4}\right)\right]$$

$$\text{with } h_1 = 2\left(1+h^2\right) \text{ and } h_2 = \left(1-\frac{1}{h_1}\right)^{2h^2-1}$$

In our case, for h=0.5/0.5=1, $F_{1+1'\to2+2'}$=0.200, and for h=0.25/0.5=1/2, $F_{1'\to2'}$=0.240.

We may also find tabulated the view factor between adjacent unequal plates in a general case, but will take advantage of the fact that, for $F_{1'\to 2+2'}$, it is the special case of a rectangular to a square plate:

Table 2. View factor between adjacent rectangle and square plate at 90º, of height H and width W, with $h\equiv W/L$.



$$F_{1\to 2}=\frac{1}{4}+\frac{1}{\pi}\left[h\arctan\left(\frac{1}{h}\right)-h_1\arctan\left(\frac{1}{h_1}\right)-\frac{h^2}{4}\ln\left(h_2\right)\right]$$

$$\text{with } h_1=\sqrt{1+h^2} \text{ and } h_2=\frac{h_1^4}{h^2\left(2+h^2\right)}$$

In our case h=0.25/0.5=1/2 and $F_{1'\to 2'+2'}=F_{1+1'\to 2'}$=0.146. We can also recover the case $F_{1+1'\to 2+2'}$=0.200 with h=1.

Now, from view-factor algebra, distribution relation is $F_{i,j+k}=F_{ij}+F_{ik}$, and reciprocity relation $A_iF_{ij}=A_jF_{ji}$, so that $F_{i+j,k}=\left(A_iF_{ik}+A_jF_{jk}\right)/\left(A_i+A_j\right)$, and in our case:

$$F_{1\to 2}=F_{1\to 2+2'}-F_{1\to 2'}=\frac{A_{2+2'}}{A_1}F_{2+2'\to 1}-\frac{A_{2'}}{A_1}F_{2'\to 1}=\frac{A_{2+2'}}{A_1}\left(F_{2+2'\to 1+1'}-F_{2+2'\to 1'}\right)-\frac{A_{2'}}{A_1}\left(F_{2'\to 1+1'}-F_{2'\to 1'}\right)$$

from which we get $F_{1\to 2}$=2·(0.200–0.146)–(2·0.146–0.241)=0.057, meaning that 5.7 % of the radiation emanating from surface 1 (by emission or reflection), will reach plate 2.

b) The required power to be applied to plate 2 in order to keep it at 20 ºC in an environment that may either be at 2.7 K or at 15 ºC.

We must apply the exitance (radiosity, J) method (see Thermal radiation network model). To establish the nodal equations we may use the electrical-analogy circuit of Fig. 2.

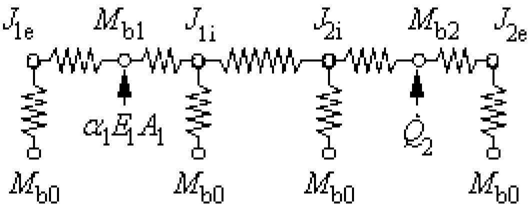


Fig. 2. Electrical-analogy circuit.

At every node (1e,b1,1i,2i,b2,2e) we set the heat-flow balance.

Node 1e (plate 1, external face node):

$$\frac{M_{b1}-J_{1e}}{\frac{1-\varepsilon_{1e}}{A_1\varepsilon_{1e}}}=\frac{J_{1e}-M_{b0}}{\frac{1}{A_1F_{1e\to 0}}} \tag{1}$$

Node b1 (blackbody node in plate 1):

$$\alpha_1 A_1 E_1=\frac{M_{b1}-J_{1e}}{\frac{1-\varepsilon_{1e}}{A_1\varepsilon_{1e}}}+\frac{M_{b1}-J_{1i}}{\frac{1-\varepsilon_{1i}}{A_1\varepsilon_{1i}}} \tag{2}$$

Node 1i (plate 1, internal face node):

$$\frac{M_{b1}-J_{1i}}{\frac{1-\varepsilon_{1i}}{A_1\varepsilon_{1i}}}=\frac{J_{1i}-M_{b0}}{\frac{1}{A_1F_{1i\to 0}}}+\frac{J_{1i}-J_{2i}}{\frac{1}{A_1F_{1i\to 2}}} \tag{3}$$

Node 2i (plate 2, internal face node):

$$\frac{J_{1i}-J_{2i}}{\frac{1}{A_1F_{1i\to 2}}}=\frac{J_{2i}-M_{b0}}{\frac{1}{A_2F_{2i\to 0}}}+\frac{J_{2i}-M_{b2}}{\frac{1-\varepsilon_{2i}}{A_2\varepsilon_{2i}}} \tag{4}$$

Node b2 (blackbody node in plate 1):

$$\frac{J_{2i}-M_{b2}}{\frac{1-\varepsilon_{2i}}{A_2\varepsilon_{2i}}}+\dot{Q}_2=\frac{M_{b2}-J_{2e}}{\frac{1-\varepsilon_{2e}}{A_2\varepsilon_{2e}}} \tag{5}$$

Node 2e (plate 1, external face node):

$$\frac{M_{b2}-J_{2e}}{\frac{1-\varepsilon_{2e}}{A_2\varepsilon_{2e}}}=\frac{J_{2e}-M_{b0}}{\frac{1}{A_2F_{2e\to 0}}} \tag{6}$$

Notice that the two first ((1) & (2)) and the two last ((5) & (6)) can be combined to skip the intermediate unknowns J_{1e} and J_{2e}:

$$\left.\begin{array}{l}\dfrac{M_{b1}-J_{1e}}{\frac{1-\varepsilon_{1e}}{A_1\varepsilon_{1e}}}=\dfrac{J_{1e}-M_{b0}}{\frac{1}{A_1F_{1e\to 0}}}\\ \alpha_1 A_1 E_1=\dfrac{M_{b1}-J_{1e}}{\frac{1-\varepsilon_{1e}}{A_1\varepsilon_{1e}}}+\dfrac{M_{b1}-J_{1i}}{\frac{1-\varepsilon_{1i}}{A_1\varepsilon_{1i}}}\end{array}\right\}\Rightarrow \alpha_1 A_1 E_1=\frac{M_{b1}-M_{b0}}{\frac{1-\varepsilon_{1e}}{A_1\varepsilon_{1e}}+\frac{1}{A_1F_{1e\to 0}}}+\frac{M_{b1}-J_{1i}}{\frac{1-\varepsilon_{1i}}{A_1\varepsilon_{1i}}} \tag{7}$$

$$\left.\begin{array}{l}\dfrac{J_{2i}-M_{b2}}{\frac{1-\varepsilon_{2i}}{A_2\varepsilon_{2i}}}+\dot{Q}_2=\dfrac{M_{b2}-J_{2e}}{\frac{1-\varepsilon_{2e}}{A_2\varepsilon_{2e}}}\\ \dfrac{M_{b2}-J_{2e}}{\frac{1-\varepsilon_{2e}}{A_2\varepsilon_{2e}}}=\dfrac{J_{2e}-M_{b0}}{\frac{1}{A_2F_{2e\to 0}}}\end{array}\right\}\Rightarrow \frac{J_{2i}-M_{b2}}{\frac{1-\varepsilon_{2i}}{A_2\varepsilon_{2i}}}+\dot{Q}_2=\frac{M_{b2}-M_{b0}}{\frac{1-\varepsilon_{2e}}{A_2\varepsilon_{2e}}+\frac{1}{A_2F_{2e\to 0}}} \tag{8}$$

Solving the four equations (7), (3), (4), (8) to find the four unknowns M_{b1}, J_{1i}, J_{2i}, and $\dot{Q}_2$ (with $\alpha_1 A_1 E_1$=0.6·0.125·800=60 W, $M_{b2}=\sigma T_2^4$=5.67·10^{-8}·293^4=418 W/m^2, $M_{b0}=\sigma T_0^4$), we get:

- For T_0=2.7 K: M_{b1}=306 W/m^2, J_{1i}=247 W/m^2$_i$, J_{2i}=216 W/m^2$_i$, and $\dot{Q}_2$=52 W.
- For T_0=288 K: M_{b1}=691 W/m^2, J_{1i}=631 W/m^2$_i$, J_{2i}=411 W/m^2$_i$, and $\dot{Q}_2$=2.6 W.

In conclusion, to keep plate 2 at 20 ºC in the T_0=2.7 K environment, we have to apply to it a power of 52 W, whereas in a T_0=288 K environment, we have to apply 2.6 W only.

c) Temperature of plate 1 in each case.

Although plate 2 is kept at 293 K in both cases, plate 1 attains $T_1=(M_{b1}/\sigma)^{1/4}$=271 K (−2 ºC) in the former case, and $T_1=(M_{b1}/\sigma)^{1/4}$=332 K (59 ºC) in the latter.

Back to Spacecraft thermal control
Back to Heat and mass transfer
Back to Thermodynamics