

TWO NON-ADJACENT PERPENDICULAR PLATES (DOS PLACAS PERPENDICULARES SEPARADAS)

Statement

Consider the radiative coupling between two equal-size plates (1 and 2 in Fig. 1) of $0.25 \cdot 0.50 \text{ m}^2$. The plates are separated 0.25 m from the common line of their corresponding planes. Plate 1 has $\alpha=0,6$ y $\varepsilon=0,8$ at both faces, and gets 800 W/m^2 normal to its external face (from a collimated source), whereas plate 2 has $\varepsilon=0,5$ at its upper face, and $\varepsilon=0,8$ at the lower one. Find:

- All the view factors involved.
- The required power to be applied to plate 2 in order to keep it at 20°C in an environment that may either be at 2.7 K or at 15°C .
- Temperature of plate 1 in each case.

Enunciado

Considérese la radiación térmica entre las placas 1 y 2 de la Fig. 1. Ambas placas son de $0,25 \cdot 0,50 \text{ m}^2$, y están desplazadas 0,25 m respecto a la línea de intersección de los planos que las contienen. La placa 1 tiene $\alpha=0,6$ y $\varepsilon=0,8$ por ambas caras, y por su cara exterior absorbe 800 W/m^2 de radiación perpendicular (proveniente de una fuente colimada), mientras que la placa 2 tiene $\varepsilon=0,5$ por la cara superior y $\varepsilon=0,8$ por la inferior. Se pide:

- Factores geométricos.
- Potencia necesaria para mantener la placa 2 a 20°C en un entorno a $2,7 \text{ K}$, y en un entorno a 15°C .
- Temperaturas de la placa 1 en cada caso.

Solution

Note. A variation of this problem is solved [aside](#).

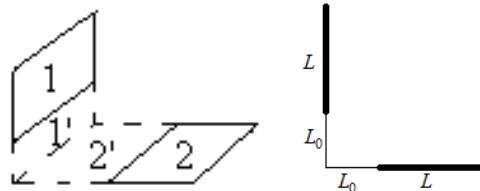
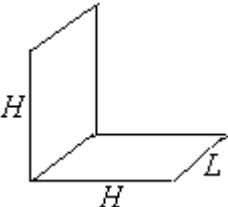


Fig. 1. Geometry.

- All the view factors involved.

We may find [tabulated](#) the view factor between adjacent equal plates, i.e. $F_{1' \rightarrow 2'}$ and $F_{1+1' \rightarrow 2+2'}$:

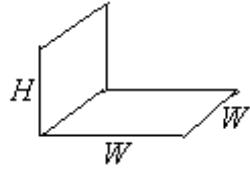
Table 1. View factor between adjacent equal rectangles at 90° , of height H and width L , with $h \equiv H/L$.

	$F_{1 \rightarrow 2} = \frac{1}{\pi} \left[2 \arctan \left(\frac{1}{h} \right) - \sqrt{2} \arctan \left(\frac{1}{\sqrt{2}h} \right) + \frac{1}{4h} \ln \left(\frac{h_1 h_2}{4} \right) \right]$ <p style="text-align: right;">with $h_1 = 2(1+h^2)$ and $h_2 = \left(1 - \frac{1}{h_1}\right)^{2h^2-1}$</p>
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In our case, for $h=0.5/0.5=1$, $F_{1+1' \rightarrow 2+2'}=0.200$, and for $h=0.25/0.5=1/2$, $F_{1 \rightarrow 2}=0.240$.

We may also find tabulated the view factor between adjacent unequal plates in a general case, but will take advantage of the fact that, for $F_{1' \rightarrow 2+2'}$, it is the special case of a rectangular to a square plate:

Table 2. View factor between adjacent rectangle and square plate at 90°, of height H and width W , with $h=W/L$.



$$F_{1 \rightarrow 2} = \frac{1}{4} + \frac{1}{\pi} \left[h \arctan\left(\frac{1}{h}\right) - h_1 \arctan\left(\frac{1}{h_1}\right) - \frac{h^2}{4} \ln(h_2) \right]$$

with $h_1 = \sqrt{1+h^2}$ and $h_2 = \frac{h_1^4}{h^2(2+h^2)}$

In our case $h=0.25/0.5=1/2$ and $F_{1' \rightarrow 2+2'}=F_{1+1' \rightarrow 2'}=0.146$. We can also recover the case $F_{1+1' \rightarrow 2+2'}=0.200$ with $h=1$.

Now, from view-factor algebra, distribution relation is $F_{i,j+k} = F_{ij} + F_{ik}$, and reciprocity relation $A_i F_{ij} = A_j F_{ji}$, so that $F_{i+j,k} = (A_i F_{ik} + A_j F_{jk}) / (A_i + A_j)$, and in our case:

$$F_{1 \rightarrow 2} = F_{1 \rightarrow 2+2'} - F_{1 \rightarrow 2'} = \frac{A_{2+2'}}{A_1} F_{2+2' \rightarrow 1} - \frac{A_{2'}}{A_1} F_{2' \rightarrow 1} = \frac{A_{2+2'}}{A_1} (F_{2+2' \rightarrow 1+1'} - F_{2+2' \rightarrow 1'}) - \frac{A_{2'}}{A_1} (F_{2' \rightarrow 1+1'} - F_{2' \rightarrow 1'})$$

from which we get $F_{1 \rightarrow 2}=2 \cdot (0.200-0.146)-(2 \cdot 0.146-0.241)=0.057$, meaning that 5.7 % of the radiation emanating from surface 1 (by emission or reflection), will reach plate 2.

- b) The required power to be applied to plate 2 in order to keep it at 20 °C in an environment that may either be at 2.7 K or at 15 °C.

We must apply the exitance (radiosity, J) method (see [Thermal radiation network model](#)). To establish the nodal equations we may use the electrical-analogy circuit of Fig. 2.

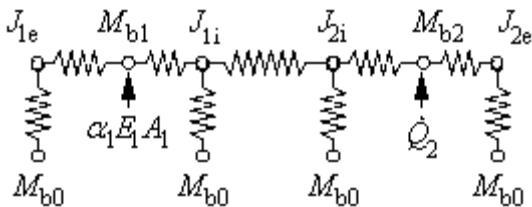


Fig. 2. Electrical-analogy circuit.

At every node (1e,b1,1i,2i,b2,2e) we set the heat-flow balance.

Node 1e (plate 1, external face node):

$$\frac{\frac{M_{b1} - J_{1e}}{1 - \varepsilon_{1e}}}{\frac{A_1 \varepsilon_{1e}}{A_1 F_{1e \rightarrow 0}}} = \frac{\frac{J_{1e} - M_{b0}}{1}}{\frac{1}{A_1 F_{1e \rightarrow 0}}} \quad (1)$$

Node b1 (blackbody node in plate 1):

$$\alpha_1 A_1 E_1 = \frac{\frac{M_{b1} - J_{1e}}{1 - \varepsilon_{1e}}}{\frac{A_1 \varepsilon_{1e}}{A_1 F_{1e \rightarrow 0}}} + \frac{\frac{M_{b1} - J_{1i}}{1 - \varepsilon_{1i}}}{\frac{A_1 \varepsilon_{1i}}{A_1 F_{1i \rightarrow 0}}} \quad (2)$$

Node 1i (plate 1, internal face node):

$$\frac{\frac{M_{b1} - J_{1i}}{1 - \varepsilon_{1i}}}{\frac{A_1 \varepsilon_{1i}}{A_1 F_{1i \rightarrow 0}}} = \frac{\frac{J_{1i} - M_{b0}}{1}}{\frac{1}{A_1 F_{1i \rightarrow 0}}} + \frac{\frac{J_{1i} - J_{2i}}{1}}{\frac{1}{A_1 F_{1i \rightarrow 2}}} \quad (3)$$

Node 2i (plate 2, internal face node):

$$\frac{\frac{J_{1i} - J_{2i}}{1}}{\frac{1}{A_1 F_{1i \rightarrow 2}}} = \frac{\frac{J_{2i} - M_{b0}}{1}}{\frac{1}{A_2 F_{2i \rightarrow 0}}} + \frac{\frac{J_{2i} - M_{b2}}{1 - \varepsilon_{2i}}}{\frac{A_2 \varepsilon_{2i}}{A_2 F_{2i \rightarrow 2}}} \quad (4)$$

Node b2 (blackbody node in plate 1):

$$\frac{\frac{J_{2i} - M_{b2}}{1 - \varepsilon_{2i}} + \dot{Q}_2}{\frac{A_2 \varepsilon_{2i}}{A_2 F_{2i \rightarrow 0}}} = \frac{\frac{M_{b2} - J_{2e}}{1 - \varepsilon_{2e}}}{\frac{A_2 \varepsilon_{2e}}{A_2 F_{2e \rightarrow 0}}} \quad (5)$$

Node 2e (plate 1, external face node):

$$\frac{\frac{M_{b2} - J_{2e}}{1 - \varepsilon_{2e}}}{\frac{A_2 \varepsilon_{2e}}{A_2 F_{2e \rightarrow 0}}} = \frac{\frac{J_{2e} - M_{b0}}{1}}{\frac{1}{A_2 F_{2e \rightarrow 0}}} \quad (6)$$

Notice that the two first ((1) & (2)) and the two last ((5) & (6)) can be combined to skip the intermediate unknowns J_{1e} and J_{2e} :

$$\left. \begin{aligned} \frac{\frac{M_{b1} - J_{1e}}{1 - \varepsilon_{1e}}}{\frac{A_1 \varepsilon_{1e}}{A_1 F_{1e \rightarrow 0}}} &= \frac{\frac{J_{1e} - M_{b0}}{1}}{\frac{1}{A_1 F_{1e \rightarrow 0}}} \\ \alpha_1 A_1 E_1 &= \frac{\frac{M_{b1} - J_{1e}}{1 - \varepsilon_{1e}}}{\frac{A_1 \varepsilon_{1e}}{A_1 F_{1e \rightarrow 0}}} + \frac{\frac{M_{b1} - J_{1i}}{1 - \varepsilon_{1i}}}{\frac{A_1 \varepsilon_{1i}}{A_1 F_{1i \rightarrow 0}}} \end{aligned} \right\} \Rightarrow \alpha_1 A_1 E_1 = \frac{\frac{M_{b1} - M_{b0}}{1 - \varepsilon_{1e}}}{\frac{A_1 \varepsilon_{1e}}{A_1 F_{1e \rightarrow 0}}} + \frac{\frac{M_{b1} - J_{1i}}{1 - \varepsilon_{1i}}}{\frac{A_1 \varepsilon_{1i}}{A_1 F_{1i \rightarrow 0}}} \quad (7)$$

$$\left. \begin{aligned} \frac{\frac{J_{2i} - M_{b2}}{1 - \varepsilon_{2i}} + \dot{Q}_2}{\frac{A_2 \varepsilon_{2i}}{A_2 F_{2i \rightarrow 0}}} &= \frac{\frac{M_{b2} - J_{2e}}{1 - \varepsilon_{2e}}}{\frac{A_2 \varepsilon_{2e}}{A_2 F_{2e \rightarrow 0}}} \\ \frac{\frac{M_{b2} - J_{2e}}{1 - \varepsilon_{2e}}}{\frac{A_2 \varepsilon_{2e}}{A_2 F_{2e \rightarrow 0}}} &= \frac{\frac{J_{2e} - M_{b0}}{1}}{\frac{1}{A_2 F_{2e \rightarrow 0}}} \end{aligned} \right\} \Rightarrow \frac{\frac{J_{2i} - M_{b2}}{1 - \varepsilon_{2i}} + \dot{Q}_2}{\frac{A_2 \varepsilon_{2i}}{A_2 F_{2i \rightarrow 0}}} = \frac{\frac{M_{b2} - M_{b0}}{1 - \varepsilon_{2e}}}{\frac{A_2 \varepsilon_{2e}}{A_2 F_{2e \rightarrow 0}}} \quad (8)$$

Solving the four equations (7), (3), (4), (8) to find the four unknowns M_{b1} , J_{1i} , J_{2i} , and \dot{Q}_2 (with $\alpha_1 A_1 E_1 = 0.6 \cdot 0.125 \cdot 800 = 60 \text{ W}$, $M_{b2} = \sigma T_2^4 = 5.67 \cdot 10^{-8} \cdot 293^4 = 418 \text{ W/m}^2$, $M_{b0} = \sigma T_0^4$), we get:

- For $T_0 = 2.7 \text{ K}$: $M_{b1} = 306 \text{ W/m}^2$, $J_{1i} = 247 \text{ W/m}^2$, $J_{2i} = 216 \text{ W/m}^2$, and $\dot{Q}_2 = 52 \text{ W}$.
- For $T_0 = 288 \text{ K}$: $M_{b1} = 691 \text{ W/m}^2$, $J_{1i} = 631 \text{ W/m}^2$, $J_{2i} = 411 \text{ W/m}^2$, and $\dot{Q}_2 = 2.6 \text{ W}$.

In conclusion, to keep plate 2 at $20 \text{ }^\circ\text{C}$ in the $T_0 = 2.7 \text{ K}$ environment, we have to apply to it a power of 52 W , whereas in a $T_0 = 288 \text{ K}$ environment, we have to apply 2.6 W only.

c) Temperature of plate 1 in each case.

Although plate 2 is kept at 293 K in both cases, plate 1 attains $T_1 = (M_{b1}/\sigma)^{1/4} = 271 \text{ K}$ ($-2 \text{ }^\circ\text{C}$) in the former case, and $T_1 = (M_{b1}/\sigma)^{1/4} = 332 \text{ K}$ ($59 \text{ }^\circ\text{C}$) in the latter.

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