TILTED PLATE IN MARTIAN ORBIT (PLACA INCLINADA EN ÓRBITA MARCIANA)

Statement

Consider a panel of 1·0.5·0.01 m^3 deployed from a spacecraft orbiting Mars (at 1.5 AU from the Sun) at the subsolar position and 300 km altitude, with its face-normal tilted 30º to sun rays. Neglect the effects of other parts of the spacecraft, and assume the panel is painted black on the face looking down the planet and white on the other face; take for the bulk properties of the panel ρ=50 kg/m^3, c=1000 J/(kg·K), and k=0.01 W/(m·K). Find:

a) The solar irradiance and the power absorbed from the Sun.
b) The heat loads from the planet.
c) The power emitted by the plate, assuming the whole plate is at temperature T_0, and in the case of different temperatures at each face, T_1 and T_2.
d) The steady value of T_0, T_1 and T_2 under the above conditions.
e) The temperature evolution after entering into eclipse (assuming the above T_0 as initial state).

Enunciado

Considérese un panel of 1·0.5·0.01 m^3 desplegado de un satélite orbitando Marte (a 1.5 ua del Sol), a 300 km de altitud sobre el punto subsolar, con la normal a sus caras formando 30º con los rayos solares. Despreciar el efecto de otras partes del satélite, y suponer que la cara iluminada es blanca y lo otra negra. Las propiedades volumétricas del panel son ρ=50 kg/m^3, c=1000 J/(kg·K), y k=0.01 W/(m·K). Se pide:

a) La irradiancia solar y la potencia solar absorbida.
b) Las cargas térmicas provenientes del planeta.
c) La potencia que emite la placa, suponiendo que está a temperatura uniforme, T_0, y en el caso de que las caras estén a diferente temperatura, T_1 y T_2.
d) Los valores de T_0, T_1 y T_2 si la placa estuviese en estado estacionario.
e) La evolución de la temperatura al entrar en eclipse (supóngase como estado inicial la T_0 anterior).

Solution

a) The solar irradiance and the power absorbed from the Sun.

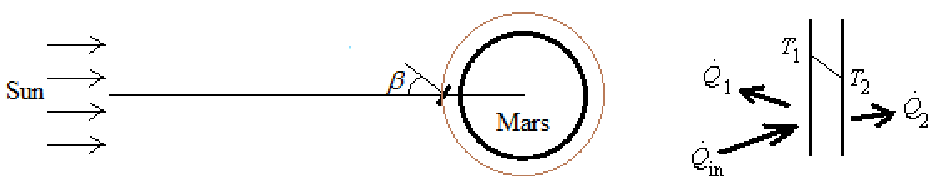

Fig. 1. A tilted plate in a Martian orbit (the plate normal is β=30º off the Sun direction), and thermal sketch.

Let '1' be the white (sunlit) face, and '2' the black face at the rear; we take their thermo-optical data from the Table. Mars orbit around the Sun has a mean radius of $R_{s\text{-}p}$=1.5 ua (230·10^9 m, with 249·10^9 m at aphelion and 207·10^9 m at perihelion), so that $E=E_0(R_{s\text{-}e}/R)^2$=1360·(1/1.5)^2=604 W/m^2 (590 W/m^2 from data Tables). Notice that Mars orbit is more eccentric than Earth's, and solar irradiance is E=714 W/m^2 at Mars perihelion and 494 W/m^2 at aphelion.

We only take into account the two main sides in the plate (edges are neglected). The face area is A=1·0.5=0.5 m^2, but the frontal area to sun-shine is $A\cos\beta$=0.5·cos30°=0.43 m^2, and the absorptance for a white paint is 0.20 (from data Tables); thence, the absorbed solar power (direct), $\dot{Q}_s$, is:

$$\dot{Q}_s = \alpha E A \cos\beta = 0.2\cdot 604\cdot 0.5\cdot \cos 30^\circ = 52 \text{ W}.$$

b) The heat loads from the planet.

Two kinds of heat loads come from the planet: reflected solar radiation (albedo contribution, $\dot{Q}_a$, maximum at the sub-solar point), and emitted infrared radiation, $\dot{Q}_p$. Because of the tilting, both sides of the detached plate get radiation from the planet. In both cases:

$$\dot{Q}_{ia} = \alpha_i A F_{ip} \rho_p E \; .$$
$$\dot{Q}_{ip} = \varepsilon_i A F_{ip} \varepsilon_p \sigma T_p^4 \; .$$

but the view factors for each face (i=1,2) to the planet (p) are different. Relative altitude is h=H/R=300/3400=0.088. Mars' surface mean surface temperature T_p=217 K, albedo ρ_p=0.15, and emissivity ε_p=0.95, can be found in Tables. From view factor Tables we get F_{1p}=0.002 (the sunlit face of the plate can see part of the planet), and F_{2p}=0.73, so that:

$$\dot{Q}_{1a} = \alpha_1 A F_{1p} \rho_p E = 0.2\cdot 0.5\cdot 0.002\cdot 0.15\cdot 604 = 0.02 \text{ W}$$
$$\dot{Q}_{2a} = \alpha_2 A F_{2p} \rho_p E = 0.95\cdot 0.5\cdot 0.73\cdot 0.15\cdot 604 = 32 \text{ W} \; .$$
$$\dot{Q}_{1p} = \varepsilon_1 A F_{1p} \varepsilon_p \sigma T_p^4 = 0.85\cdot 0.5\cdot 0.002\cdot 0.95\cdot 5.67\cdot 10^{-8}\cdot 217^4 = 0.1 \text{ W}$$
$$\dot{Q}_{2p} = \varepsilon_2 A F_{2p} \varepsilon_p \sigma T_p^4 = 0.90\cdot 0.5\cdot 0.73\cdot 0.95\cdot 5.67\cdot 10^{-8}\cdot 217^4 = 39 \text{ W} \; .$$

c) The power emitted by the plate, assuming the whole plate is at temperature T_0, and in the case of different temperatures at each face, T_1 and T_2.

For the isothermal case $\dot{Q}_{out} = A(\varepsilon_1+\varepsilon_2)\sigma T_0^4$, with A=1·0.5=0.5 m^2, since now both sides emit the same, whereas for the two-temperature case, $\dot{Q}_{out} = A\varepsilon_1\sigma T_1^4 + A\varepsilon_2\sigma T_2^4$, with emissivity of 0.85 for a white paint and 0.90 for a black paint.

d) The steady value of T_0, T_1 and T_2.

In both cases, at steady state, $\dot{Q}_{in} = \dot{Q}_{out}$. For the isothermal case (one node problem),

$$\dot{Q}_{1+2,s} + \dot{Q}_{1+2,a} + \dot{Q}_{1+2,p} = \dot{Q}_{1+2,\infty} \quad \rightarrow \quad 52+32+39 = 0.5\cdot(0.85+0.90)\cdot 5.67\cdot 10^{-8}\cdot T_0^4$$
$$\rightarrow \quad T_0 = 223 \text{ K } (-50\text{ °C}) \; .$$

whereas for the two-temperature case, an additional equation is needed to relate T_1 and T_2, which is the conductive coupling $kA(T_1-T_2)/L = A\varepsilon\sigma T_2^4$, forming a non-linear system of two equations:

$$\dot{Q}_{1s} + \dot{Q}_{1a} + \dot{Q}_{1p} = \dot{Q}_{1\infty} + kA\frac{T_1-T_2}{\delta} \quad \rightarrow \quad 52+0.02+0.1 = 0.5\cdot(0.85+0.90)\cdot 5.67\cdot 10^{-8}\cdot T_1^4 + 0.1\cdot 0.5\frac{T_1-T_2}{0.01}$$
$$\dot{Q}_{2s} + \dot{Q}_{2a} + \dot{Q}_{2p} + kA\frac{T_1-T_2}{\delta} = \dot{Q}_{2\infty} \quad \rightarrow \quad 0+32+39+0.1\cdot 0.5\frac{T_1-T_2}{0.01} = 0.5\cdot(0.85+0.90)\cdot 5.67\cdot 10^{-8}\cdot T_2^4 \; .$$

easily solved numerically to yield T_1=220 K and T_2=227 K.

e) The temperature evolution after entering into eclipse with the above initial state.

Simplifying to the one-temperature case, the cooling is governed to a first approximation by $mc\mathrm{d}T/\mathrm{d}t=-2A\varepsilon\sigma T^4$, since solar and albedo inputs are zero, and planet IR is small (39 W against 52+32+39=123 W. This simplification allows a simple solution, $T(t)=T_0[1/(1+t/\tau)^{1/3}]$, where the relaxation time is $\tau=mc/(6A\varepsilon\sigma T_0^3)$. Substitution of the panel mass, $m=\rho LA$=50·0.01·0.5=0.25 kg, and the rest of the data finally yields τ=150 s; $T(t)$ is plotted in Fig. 2 during a typical eclipse duration of 42 minutes in a 114 minutes' orbit period.

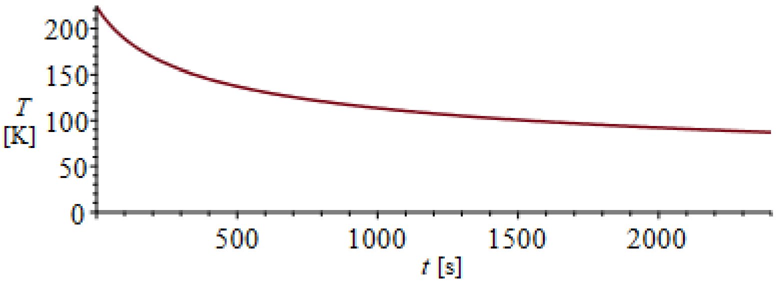

Fig. 2. Temperature evolution (one-node model) after entering into eclipse (until eclipse ends).

We see that the cooling becomes less effective as time passes (at lower temperatures), so that at the end of a 40 min eclipse duration the plate temperature has only dropped from 223 K to 86 K, and this is without accounting for the infrared heating from Mars. Orbit period around Mars depend on altitude, with around 1.9 h for low orbits (Phobos, at 6000 km altitude, has a period of 7.7 h, while Deimos, at 20 000 km, has T=30 h and is just outside synchronous orbit).

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