Isidoro Martínez

HEAT CONDUCTION IN AN ELECTRONICS PLATE

Statement

Consider an instrument plate in a satellite, consisting of two layers of CFRP of 2 mm thick, and a layer of solid foam (F) of 12 mm inside which elements of the electric battery are housed in a modular way. Each plate module measures 300×80×16 mm^3, and holds one Li-ion battery module (B) embedded on a centred slit of 140×70×6 mm^3 on one side of the foam (F); the effect of connections and auxiliary circuits are neglected. The following simplified thermal model is to be considered (the properties of the materials are given in Table 1). The battery element, in the most critical case, dissipates 1 W during a 30 minutes period, being idle for the other 60 minutes of the cycle. We only consider heat transfer by conduction to one of the short ends of the module, which is assumed to remain at 300 K (the other three sides of symmetry can be assumed). In particular:

a) Solve the steady heat-conduction problem (1 W) from the battery element (B, to be considered in this point as isothermal) to the plate edge (T_b=300 K), through the two parallel paths: 1) direct path along the contact face; and 2) indirectly through the 6 mm foam and then along the other sheet of CFRP.
b) Calculate the admissible thermal load of equipment to be mounted on the plate so that the plate temperature does not exceed 75 °C.
c) Solve analytically the one-dimensional steady heat-conduction problem along the layer of CFRP alone (excluding battery) with 1 W uniformly distributed over the entire CFRP layer.
d) Solve analytically the one-dimensional steady heat-conduction problem along the layer of CFRP alone (excluding battery), but now with energy dissipation evenly distributed only in the transverse direction of the module (80 mm).
e) Solve analytically the one-dimensional steady heat-conduction problem along the combined layer of CFRP and battery (but not through the foam).
f) Solve numerically the one-dimensional unsteady heat-conduction problem with the temporal variation of the energy dissipation given.
g) Solve numerically the problem of two-dimensional transient conduction from the isothermal state to the steady state of 1 W dissipation at the battery element.

Table 1. Materials properties.

	ρ [kg/m^3]	c [J/(kg·K)]	k [W/(m·K)]
CFRP	1500	840	30 (parallel) 1,25 (perpend.)
F	75	1200	0.03
B	2400	900	35 (parallel) 1,5 (perpend.)

En un satélite se piensa usar una bandeja porta-instrumentos construida en panel compuesto, formado por dos capas de CFRP de 2 mm de espesor, y una capa de espuma sólida (F) de 12 mm de espesor en cuyo interior van alojados los elementos de la batería eléctrica. Se va a estudiar el comportamiento térmico de uno de los módulos laterales de la bandeja, de dimensiones 300×80×16 mm^3. En una de las caras de F hay una hendidura centrada que aloja un elemento de batería (B) de Li-ion, de 140×70×6 mm^3; el efecto de las conexiones y circuitos auxiliares de la batería no se tienen en cuenta. Se va a estudiar el siguiente modelo térmico simplificado (las propiedades de los materiales se dan en la Tabla 1): el elemento de batería, en el caso más crítico, disipa 1 W durante 30 minutos y está inactivo los siguientes 60 minutos del ciclo. Solo se va a considerar la transmisión de calor por conducción a uno de los extremos cortos del módulo, que se supondrá que permanece a 300 K (los otros tres lados se pueden suponer de simetría). En particular, se pide:

a) Resolver el problema de conducción estacionaria (1 W) desde el elemento de batería (B, que se considerará isotermo en este apartado), hasta el borde de la placa (a T_b=300 K), a través de los dos caminos paralelos: 1) el directo, a lo largo de la cara en contacto; y 2) el indirecto, a través de los 6 mm de espuma y luego a lo largo de la otra lámina de CFRP.
b) Calcular la carga térmica admisible de los equipos que fueran montados sobre la bandeja, para que la temperatura de esta no supere los 75 ºC.
c) Resolver analíticamente el problema de conducción unidimensional estacionaria a lo largo de la capa de CFRP (sin contar la batería), con los 1 W distribuidos uniformemente sobre toda la cara del módulo.
d) Resolver analíticamente el problema de conducción unidimensional estacionaria a lo largo de la capa de CFRP (sin contar la batería), pero ahora con la disipación de energía distribuida uniformemente sólo en la dirección transversal del módulo (los 80 mm).
e) Resolver analíticamente el problema de conducción unidimensional estacionaria a lo largo de la capa de CFRP contabilizando también la conducción en la batería (pero no a través de la espuma).
f) Resolver numéricamente el problema de conducción unidimensional no estacionaria con la variación temporal de la disipación de energía dada.
g) Resolver numéricamente el problema de conducción bidimensional transitorio desde el estado isotermo hasta el estacionario con 1 W disipándose en la batería.

Solution

a) Solve the steady heat-conduction problem (1 W) from the battery element (B, to be considered in this point as isothermal) to the plate edge (T_b=300 K), through the two parallel paths: 1) direct path along the contact face; and 2) indirectly through the 6 mm foam and then along the other sheet of CFRP.

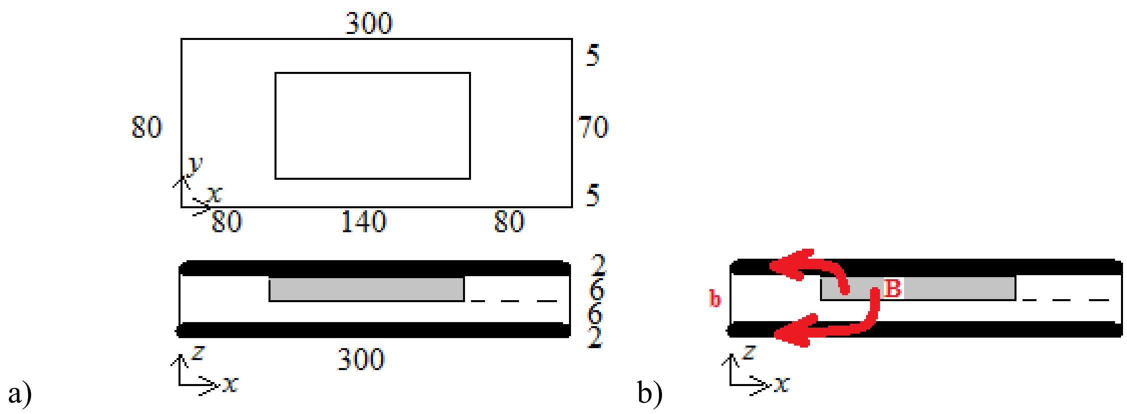


Fig. 1. a) Top view and profile of the plate element with the embedded battery element, with dimensions (in mm). b) Heat paths from battery (B) to base (b).

There are two heat paths in parallel:

1. Direct path, along the upper layer of CFRP, with a thermal resistance $R_1 \equiv \Delta T/Q_1 = L_{80}/(k_C A_C) = L_{80}/(k_C L_y L_{zC}) = 0.08/(30\cdot 0.08\cdot 0.002) = 16.7$ K/W; i.e. through the CFRP layer (2 mm thick by 80 mm wide) a fraction of the total heat, $Q_1 < Q = 1$ W, flows from the battery at T_B (which can be considered to be at x=0.080 m because of its large thermal conductance) to the base temperature T_b=300 K at x=0. Notice that we take the 80 mm width for the heat path, in spite of the 70 mm width of the battery module (the CFRP temperature in the 5 mm edges are not expected to have significant transversal gradients).
2. Indirect path, with two stages in series: one transversal across the remaining 6 mm foam thickness, followed by the heat flow along the opposite CFRP layer. The thermal resistance of this combined indirect path is $R_2 = R_{2F} + R_{2C} = L_{zF2}/(k_F A_{Fp}) + L_{80}/(k_C A_C) = 0.006/(0.03\cdot 0.08\cdot 0.14) + 16.7 = 17.9 + 16.7 = 34.5$ K/W. Notice that we take the 80 mm width for the heat path, for consistency.

The global thermal resistance, R, of the two paths (direct and indirect), being in parallel, is obtained from $1/R = 1/R_1 + 1/R_2$, what yields R=11.2 K/W, from which we obtained the battery temperature, $T_B = T_b + QR = 300 + 1\cdot 11.2 = 311.2$ ºC, and the share of heat flows: Q_1=0.67 W and Q_2=0.33 W. The temperature jump across the 6 mm foam under the battery is $\Delta T_{2F} = Q_2 R_{2F} = 0.33\cdot 17.9 = 5.9$ ºC, is representative of the mid-point jump (i.e. at $x = L_x/2$).

b) Calculate the admissible thermal load of equipment to be mounted on the plate so that the plate temperature does not exceed 75 °C.

It will depend on which side, and at what distance from the edge, the equipment is installed. If installed centred on the face in contact with the battery, the problem is solved by setting T_B=75 ºC above, and calculating the total heat flow as before, resulting in $Q = \Delta T/R = (75+273-300)/11.2 = 4.3$ W in total (i.e. 3,3 W plus the 1 W battery). If installed centred but on the opposite side, a lower heat load would be admissible, as it can be easily seen by considering only the direct path heat resistance, which now it would be $R_1' \equiv \Delta T/Q = L_{150}/(k_C A_C) = 0.150/(30\cdot 0.08\cdot 0.002) = 31.3$ K/W and would yield $Q = \Delta T/R = (75+273-300)/31.3 = 1.5$ W instead of the previous 3.3 W. The explanation is that the battery material helps to carry away the heat load best if there is no insulation between.

b) Solve analytically the one-dimensional steady heat-conduction problem along the layer of CFRP alone (excluding battery) with 1 W uniformly distributed over the entire CFRP layer.

The 1D-heat-conduction equation with a volumetric heat source ϕ, $\rho c\partial T/\partial t=k\partial^2 T/\partial t^2+\phi$, can be integrated at the steady state to yield $T(x)=a+xb-x^2\phi/(2k)$, which, with $\phi=W/(L_xL_yL_{zC})=10^9/(300\cdot80\cdot2)=21$ kW/m^3, and the boundary conditions, $T(0)=a=T_0$ and $\mathrm{d}T/\mathrm{d}x|_{x=0}=b=\phi L/k$, yield $T(x)=T_0+(\phi/k)(xL-x^2/2)$, with a maximum value of $T_{\max}=T_0+\phi L^2/(2k)=300+21\cdot10^3\cdot0.3^2/(2\cdot30)=331$ K (58 ºC).

A crude estimation of the time to reach the steady state from initial conditions can be obtained by solving an energy balance with the Q=1 W employed just on heating the CFRP layer from 300 K to 331 K, i.e $t_c=mc\Delta T/Q=\rho L_xL_yL_zc\Delta T/Q=1500\cdot0.3\cdot0.08\cdot0.002\cdot840\cdot(331-300)/1=1900$ s, although this is a lower bound because it does not takes into account the heat being lost and the thermal capacity of battery and the rest.

d) Solve analytically the one-dimensional steady heat-conduction problem along the layer of CFRP alone (excluding battery), but now with energy dissipation evenly distributed only in the transverse direction of the module (80 mm).

Now, the parabolic temperature profile obtained above, is only valid for the central segment (from x_1=80 mm to x_2=220 mm), and it must be matched to two linear T-profiles at the extremes (where ϕ=0), i.e. $T_{01}(x)=a_1+xb_1$, $T_{12}(x)=a_2+xb_2-x^2\phi/(2k)$, $T_{23}(x)=T_2$ (constant because no heat flow to the right). Imposing the continuity of the T-profile, $T(x)$, at the two intermediate discontinuities, T_1 y T_2, and the continuity of the heat flow, $\dot{Q}(x)=-kA\mathrm{d}T/\mathrm{d}x$, i.e.:

At $x=L_{80}$=80 mm, $a_1+L_{80}b_1=a_2+L_{80}b_2-L_{80}^2\phi/(2k_C)$ and $-k_CAb_1=-k_CAb_2+AL_{80}\phi$.

At $x=L_{220}$=220 mm, $a_2+L_{220}b_2-L_{220}^2\phi/(2k_C)$ and $-k_CAb_2+AL_{220}\phi=0$.

The result, with $\phi=W/(L_xL_yL_{zC})=10^9/(140\cdot80\cdot2)=45$ kW/m^3, is T_1=317 K and T_2=331 K. It can be checked that any centred distribution of $\phi(x)$ yields the same maximum temperature (331 K) and the same T-gradient at x=0, even if the dissipation width were reduced to a point in the middle (at x=150 mm).

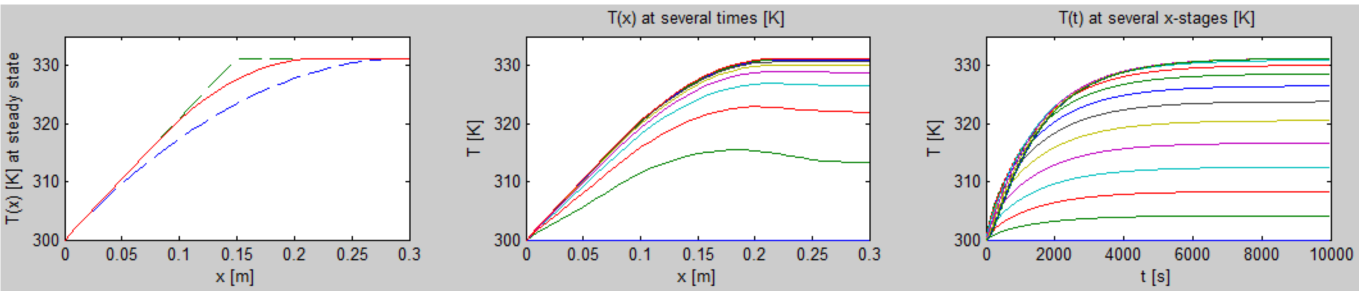


Fig. 2. a) Steady $T(x)$ profile for the present case (dissipation in the range x=80..220 mm, bold red line), and comparison with the previous case (dissipation in the whole range x=0..300 mm, dashed blue line), and with a concentrated dissipation at x=150 mm (dashed green line). b) Unsteady simulation for several time steps; notice that there is some heat flow to the right to heat the insulated part. c) Unsteady simulation at several x-stages.

e) Solve analytically the one-dimensional steady heat-conduction problem along the combined layer of CFRP and battery (but not through the foam).

The temperature profile over the battery segment will still be parabolic, but flatter because heat conductance is larger, to be merged with one straight stretch at each end. To find the $T(x)$ profile, the same conditions of continuity of $T(x)$ and $\dot{Q}(x)$, at the same interfaces (x_1=80 mm and x_2=220 mm) are imposed, but now with different properties in the stretch where the CFRP and B are in contact, i.e.:

At $x=L_{80}$=80 mm, $a_1+L_{80}b_1=a_2+L_{80}b_2-L_{80}^2\phi/(2k_{\text{eff}})$ and $-k_C A b_1=-k_{\text{eff}}A_{\text{eff}}b_2+A_{\text{eff}}L_{80}\phi$.

At $x=L_{220}$=220 mm, $a_2+L_{220}b_2-L_{220}^2\phi/(2k_{\text{eff}})$ and $-k_{\text{eff}}A_{\text{eff}}b_2+A_{\text{eff}}L_{220}\phi=0$.

If we take the whole cross-section area, $A_{\text{eff}}=A_C+A_B$, to define the effective conductivity, k_{eff}, then $k_{\text{eff}}=(k_C A_C+k_B A_B)/(A_C+A_B)=(k_C L_{zC}+k_B L_{zB})/(L_{zC}+L_{zB})=(30\cdot 2+35\cdot 6)/(2+6)=34$ W/(m·K), and the result, with $\phi=W/(L_x L_y(L_{zC}+L_{zB}))=10^9/(140\cdot 80\cdot(2+6))=11$ kW/m^3, is now T_1=317 K and T_2=320 K (instead of the previous T_2=320 K). The same result would have been obtained if we had used the original CFRP thickness to define $A_{\text{eff}}=A_C$, and then $k_{\text{eff}}=k_C+k_B L_{zB}/L_{zC}=30+35\cdot 6/2=134$ W/(m·K), and $\phi=W/(L_x L_y L_{zC})$= $10^9/(140\cdot 80\cdot 2)=45$ kW/m^3.

An estimation of the required time to reach the steady state, similar to the one made above but changing to the new thermal capacity, now yields $t_c=mc\Delta T/Q=(m_C c_C+m_B c_B)\Delta T/Q$= (0,072·840+0,141·900)·(317–300)/1=3200 s, enhancing the previous estimation of t_c=1900 s, but still a lower bound because it does not accounts yet for heat losses.

f) Solve numerically the one-dimensional unsteady heat-conduction problem with the temporal variation of the energy dissipation given.

We are going to use the finite difference method (FDM), discretizing the CFRP layer in N equal spatial segments, and finding $T(x)$ at the ends of the stretches, i.e. finding T_i at $x_i=i\Delta x$ with i=0..N and $\Delta x=L_x/N$, and all this as a function of time, i.e. discretizing $T(x,t)=T_i^j$, for j=0..M at appropriate times $t_j=j\Delta t$, to be defined below. Notice that T_i is the representative temperature around x_i (e.g. in the dashed zone in Fig. 3). For stability of the explicit time-discretization, the advancing time-step, Δt, is bound for this 1-D case to $Fo\equiv a\Delta t/(\Delta x)^2<1/2$, where $a\equiv k/(\rho c)$ is the thermal diffusivity of the material (for CFRP $a=k/(\rho c)=30/(1500\cdot 840)=24\cdot 10^{-6}$ m^2/s), and Fo is the Fourier number for a segment. The stability criterion can be explained in terms of the Second Law of Thermodynamics if we imagine the thermal relaxation of a node at T_i with the surroundings nodes (2 in one-dimensional problems) at a lower temperature; the discretized heat equation can be written as $\rho c\Delta x\Delta_t T/\Delta t=2k\Delta_x T/\Delta x$, but the Second Law forbids the temporal variation $\Delta_t T$ to surpass the spatial variation $\Delta_x T$, i.e. $\Delta_t T<\Delta_x T$, implying $2a\Delta t/(\Delta x)^2<1$ or $Fo<1/2$.

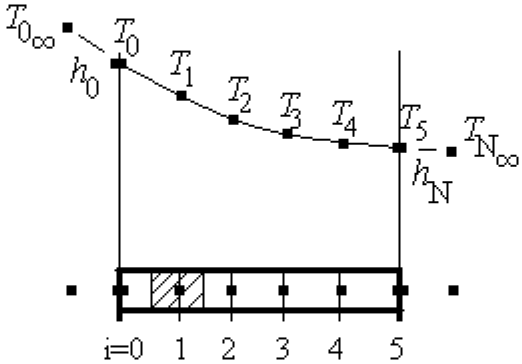


Fig. 3. Sketch of thermal nodes for numerical simulation by finite differences.

Instead of discretizing the volumetric form of the heat equation, $\rho c\partial T/\partial t=k\nabla^2 T+\phi$, we use the total 1-D heat flow, $\rho A\Delta x c\partial T/\partial t=Q_{in}+Q_{out}+\phi A\Delta x$, with $Q=-kA\partial T/\partial x$, which in a discretized form is:

$$\rho A\Delta x\, c\frac{T_i^{j+1}-T_i^j}{\Delta t}=kA\frac{T_{i+1}^j-T_i^j}{\Delta x}-kA\frac{T_i^j-T_{i-1}^j}{\Delta x}+\phi A\Delta x$$

A Matlab coding to compute $T(x,t)$ by time-steps j=1..M and spatial discretization i=1..N+1 is presented in Table 2 (just the iterative part; variable definitions must precede that).

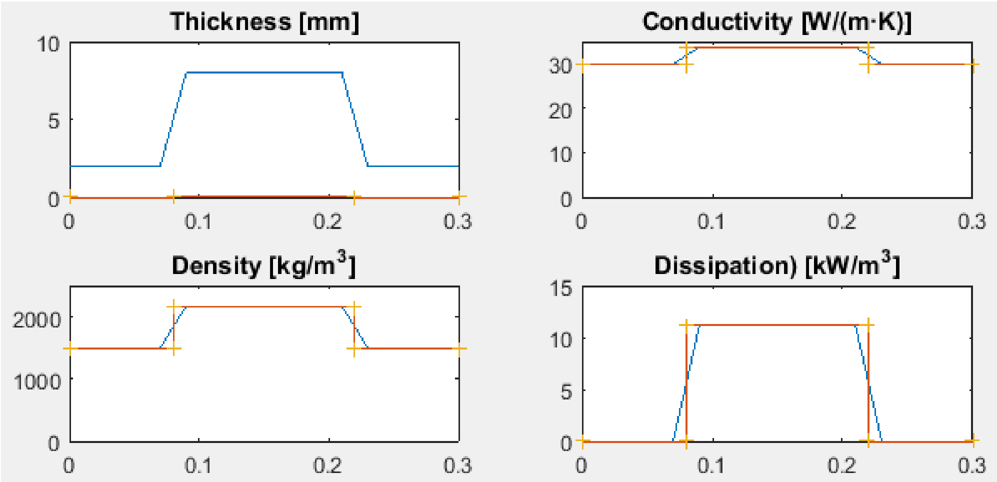
Table 2. Iterations for the numerical simulation in Matlab.

```
j=1; T(j,:)=T0;          %Initial temperature profile T(x,t)=0 (assumed uniform)
for j=2:M                %Time advance
   i=1; T(j,i)=T0;       %Left border (base) maintained at T0
   for i=2:N             %Generic spatial nodes
     T(j,i)=T(j-1,i)+(Dt/(rho*c*A))*((k*A*(T(j-1,i+1)-T(j-1,i))-k*A*(T(j-1,i)-T(j-1,i-1)))/Dx^2+phi*A);
   end
   i=N+1; T(j,i)=T(j-1,i-1);%Right border kept adiabatic
end
```

It is advisable to first solve the problem of a permanent heat release ϕ along the whole CFRP layer, or just along the CFRP section in contact with the battery (i.e. with spatial discontinuity in $\phi(x)$), to gain confidence in the numerical simulation (it is usually better to proceed by adding details to a small code than to debug a complex coding). For instance, to simulate a discontinuous $\phi(x)$, one can just change 'phi' to 'phi(i)' in the code of Table 2, provided that in previous sentences the raw discontinuous dissipation is defined in a continuous way (e.g. eps=1e-6; Xraw=[0, 0.080-eps, 0.080+eps, 0.220-eps, 0.220+eps, 0.300]; phi_raw=[0, 0, 45e3, 45e3, 0,0], and the interpolation at the equidistant stages X=linspace(0,Lx,N); phi=interp1(Xraw,phi_raw,X); is applied; 'eps' is a small-enough displacement to smooth the discontinuous function, e.g. eps=1e-6).

Considering the combined heat conduction between the sheet of CFRP and battery B, and taking the total thickness as a reference, we can calculate for the C/B set (of thickness L_{zCB}=2+6=8 mm), the effective conductivity $k_{eff}=(k_C L_{zC}+k_B L_{zB})/(L_{zC}+L_{zB})=(30\cdot 2+35*6)/(2+6)=34$ W/(m·K), the effective density $\rho_{eff}=(\rho_C L_{zC}+\rho_B L_{zB})/(L_{zC}+L_{zB})=(1500\cdot 2+2400\cdot 6)/8=2175$ kg/m^3, the effective specific heat capacity $c_{eff}=(c_C\rho_C L_{zC}+c_B\rho_B L_{zB})/(\rho_C L_{zC}+\rho_B L_{zB})=(840\cdot 1500\cdot 2+900\cdot 2400\cdot 6)/(1500\cdot 2+2400\cdot 6)=890$ J/(kg·K), and the volumetric dissipation in the battery (relative to the volume of the local set), $\phi=Q/(L_{zB}L_y L_{zCB})=1/(0{,}0140\cdot 0{,}080\cdot 0{,}006)=11$ kW/m^3, currently considered unchanged over time.

Longitudinal profiles for thickness, conductivity, density, and volumetric dissipation, are shown in Fig. 4a, and temperature profiles (spatial and temporal) in Fig. 4b. Note the slope discontinuities in $T(x)$ profile, and the fact that, in the transient state, there is a heat flow to the right (to warm the adiabatic end).



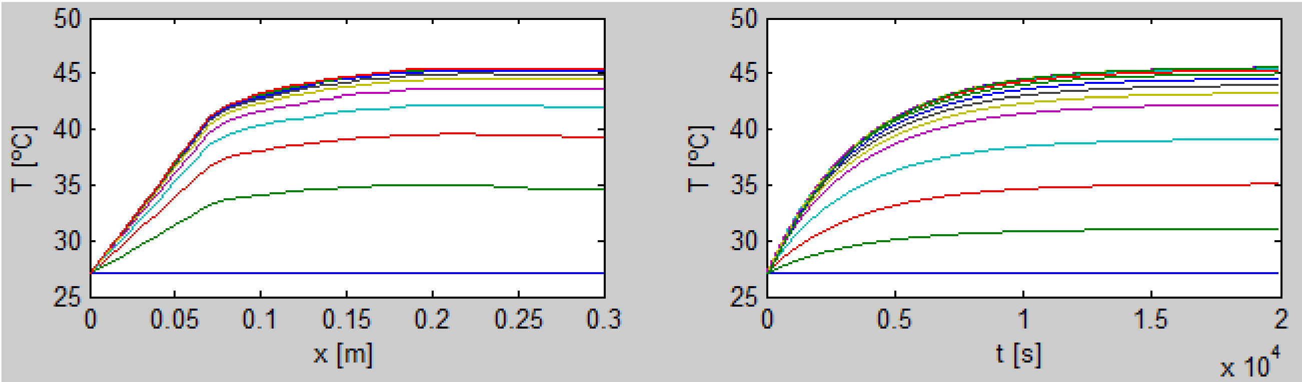


Fig. 4. a) Longitudinal profiles for thickness, conductivity, density, and volumetric dissipation; in blue the discontinuous model, and in green the continuous model discretized in 30 elements of 1 cm. b) Evolution of the temperature profile $T(x)$ for several moments, and $T(t)$ for a number of equally spaced points.

$$\rho_i c_i A_i \Delta x \frac{T_i^+ - T_i}{\Delta t} = k_{i+} A_{i+} \frac{T_{i+} - T_i}{\Delta x} - k_{i-} A_{i-} \frac{T_i - T_{i-}}{\Delta x} + \phi_i A_i \Delta x$$

$$\rightarrow T_i^+ = T_i + \frac{\Delta t}{\rho_i c_i A_i} \left[\left(k_{i+} A_{i+} \left(T_{i+} - T_i \right) - k_{i-} A_{i-} \left(T_i - T_{i-} \right) \right) \frac{1}{(\Delta x)^2} + \phi_i A_i \right]$$

Considering now that the dissipation only lasts for 30 min, with 60 min of inactivity (although this is impossible because the battery during charging also dissipate), the time profile of dissipation, $\phi(t)$, and $T_{max}(t)$ is shown in Fig. 5. As expected by the previous analysis for $\phi(t)$=const, the typical time to reach steady state was about 10 000 s (about 3 h), whereas the statement indicates that the battery dissipates just for 30 min, so that after a cycle it does not return to the initial conditions, as seen in the evolution of $T_{max}(t)$. You can also see that during the 'switch off' there is a heat flow from the adiabatic edge toward the centre, which cools faster.

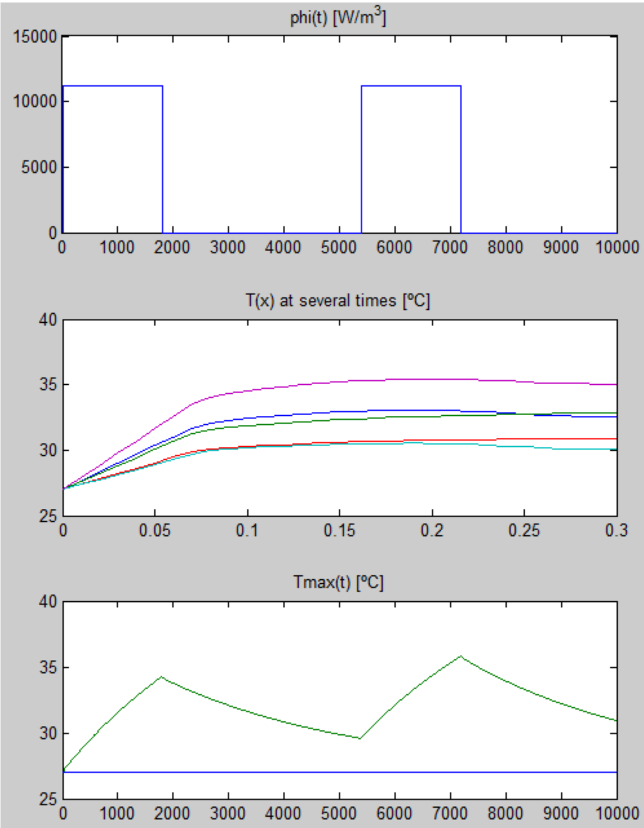


Fig. 5. Temporal dissipation profile, $\phi(t)$, spatial temperature profiles $T(x)$ for various times, and $T_{\max}(t)$.

g) Solve numerically the problem of two-dimensional transient conduction from the isothermal state to the steady state of 1 W dissipation at the battery element.

The two-dimensional planar heat equation is $\rho c\partial T/\partial t=k(\partial^2T/\partial x^{2+}\partial^2T/\partial y^2)+\phi$. When discretized by finite differences for a generic volume element dxdyL_z (where $L_z(i,j)$ is the thickness at each point, which in this case we take constant, L_y=80 mm, and with an 'extended' battery pack so that the model can be two-dimensional; i.e. we will consider its dimensions B(140,80,6) instead of B(140,70,6)), and taking into account the longitudinal variations of properties, the heat equation becomes:

$$\rho_{i,j}c_{i,j}\Delta x\Delta y L_{zi,j}\frac{T_{i,j}^{+}-T_{i,j}}{\Delta t}=k_{i+,j}\Delta y L_{zi+,j}\frac{T_{i+,j}-T_{i,j}}{\Delta x}-k_{i-,j}\Delta y L_{zi-,j}\frac{T_{i,j}-T_{i-,j}}{\Delta x}+$$
$$+k_{i,j+}\Delta x L_{zi,j+}\frac{T_{i,j+}-T_{i,j}}{\Delta y}-k_{i,j-}\Delta x L_{zi,j-}\frac{T_{i,j}-T_{i,j-}}{\Delta y}+\phi_{i,j}\Delta x\Delta y L_{zi,j}$$
$$\to T_i^{+}=T_i+\frac{\Delta t}{\rho_{i,j}c_{i,j}L_{zi,j}}\left[\begin{array}{l}\left(k_{i+,j}L_{zi+,j}\left(T_{i+,j}-T_{i,j}\right)-k_{i-,j}L_{zi-,j}\left(T_{i,j}-T_{i-,j}\right)\right)\dfrac{1}{(\Delta x)^2}+\\+\left(k_{i,j+}L_{zi,j+}\left(T_{i,j+}-T_{i,j}\right)-k_{i,j-}L_{zi,j-}\left(T_{i,j}-T_{i,j-}\right)\right)\dfrac{1}{(\Delta y)^2}+\phi_{i,j}L_{zi,j}\end{array}\right]$$

Fig. 6a shows a cut profile of the plate, with the different materials (to scale in the bottom plot), and the two-dimensional diagrams of densities $\rho(x,y)$, and specific thermal capacities c=$c(x,y)$, on the top plot with the real discontinuities, and in the bottom plot interpolated with a mesh of 30 elements in x (10 mm each) and 16 elements in y (also of 1 mm). Fig. 6b shows the two-dimensional diagrams of longitudinal thermal conductivities, $k_x(x,y)$, transversal thermal conductivities, $k_y(x,y)$, and volumetric dissipation $\phi=\phi(x,y)$, not counting for the time variation .

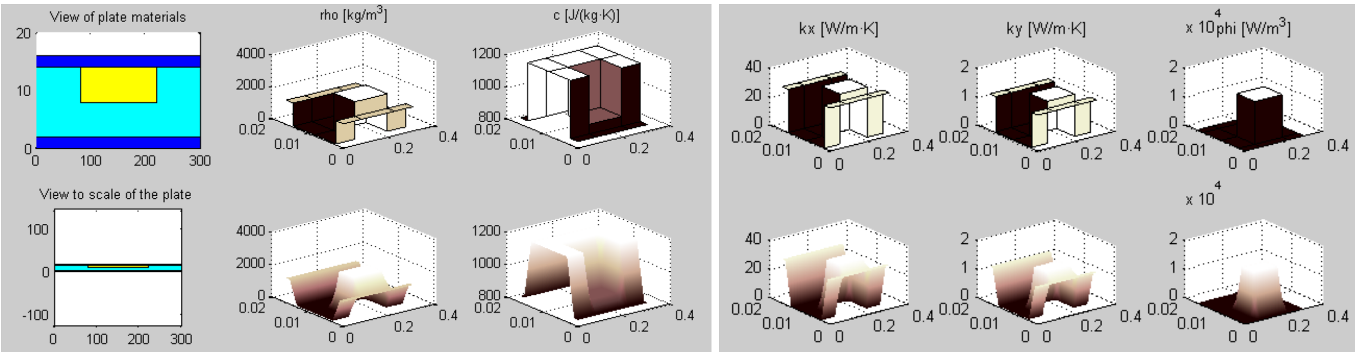


Fig. 6. a) Section of the plate, and maps of densities ρ [kg/m^3] and thermal capacities c [J/(kg·K)]. b) Maps of longitudinal thermal conductivities, $k_x(x,y)$, transverse thermal conductivities, $k_y(x,y)$, and volumetric dissipation $\phi=\phi(x,y)$.

Finally, Fig. 7 shows some details of the simulation result, and in Fig. 8 the map $T(x,y)$ at steady state (although, as already seen, if the dissipation lasts only 30 minutes (1800 s), the steady state will never be reached (about 10,000 s would be needed for that).

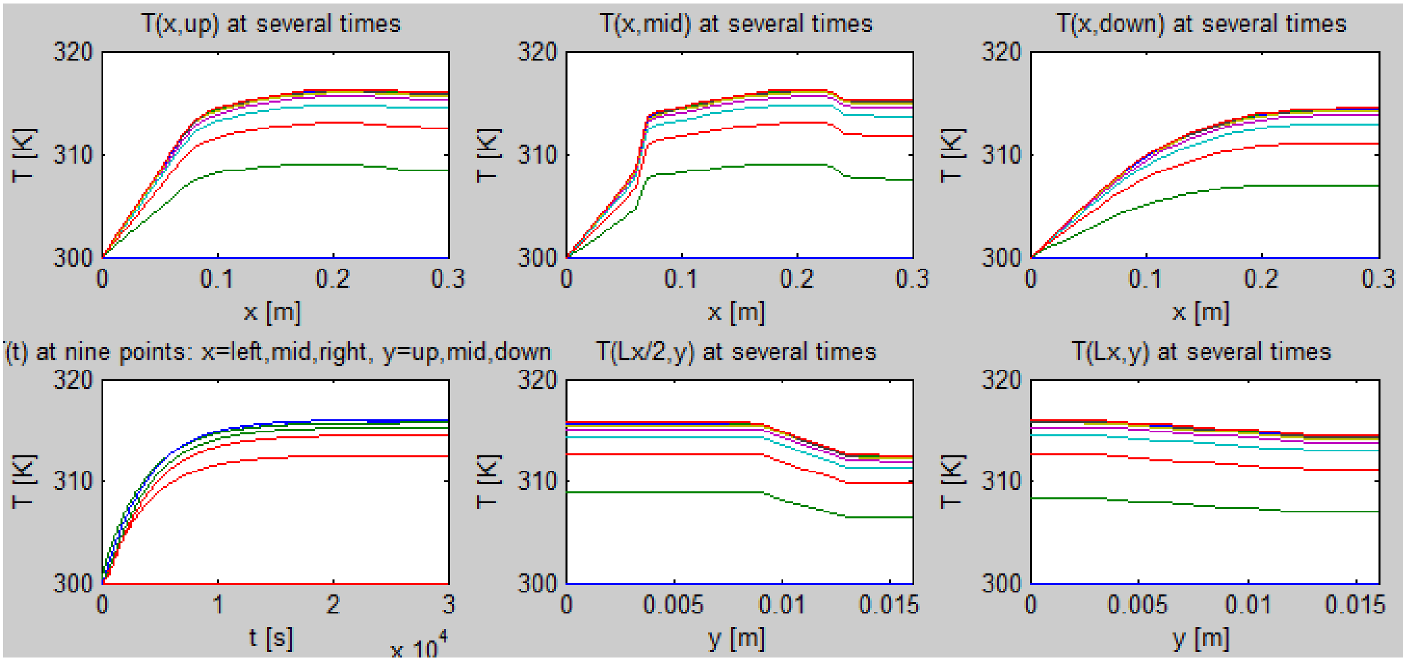


Fig. 7. From left to right and from top to bottom: a) temperature profile in the upper layer of CFRP for several times (in the stationary state, the isolated end of the plate reaches T_{max}=316 K); b) The same, but half the height of the sandwich (note that the central part, in contact with the battery, is also at 316 K, but that $T(x,L_y/2)$ decreases in the isolated end to a value almost constant because longitudinal heat transmission is negligible); c) temperature profile in the lower layer of CFRP for several times; d) Time evolution of temperatures in the 9 points of the periphery (ends and midpoints; of course, there is no change in the three end-points where 300 K were imposed; it can be seen that the steady state is reached in about 10 000 s); e) temperature profile in a section half the length (i.e. at 150 mm from the recessed edge, the temperature being 316 K in the top layer of CFRP and battery, and decreasing significantly in the 6 mm of foam, and almost nothing in the 2 mm of the CFRP-plate below); f) The same, but in the adiabatic end, i.e. at x=300 mm (note the small temperature jump between plates, due to the limited transverse heat transmission).

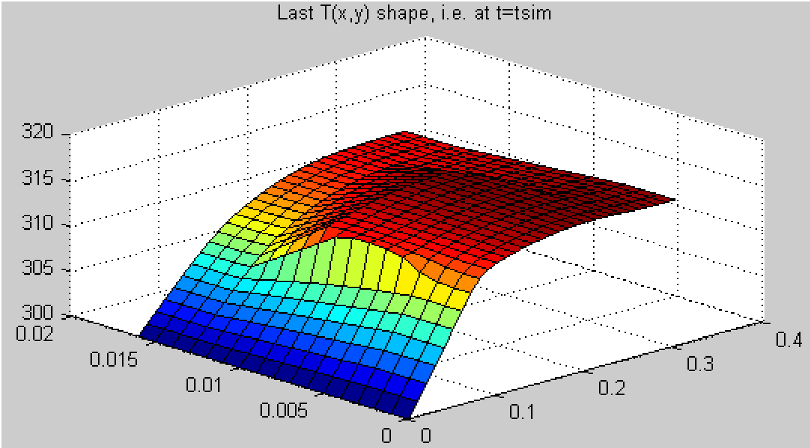


Fig. 8. Temperature map $T(x,y)$ at the steady state.

In conclusion, the maximum temperature in steady state with this detailed two-dimensional model is $T_{max}(x,y,t)$=316 K (i.e. the maximum heating is 16 K), which could have been approximated with a zero-dimensional model as explained in paragraph a).

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