TEMPERATURE OSCILLATIONS IN AN ENGINE WALL

Statement

Consider the heat transfer through the walls in a cylinder of an internal combustion engine. The inside is exposed to burning gases with an average temperature of 1200 K, oscillating ±500 K at 30 Hz (it is a small two-stroke engine running at 1800 rpm), with which a convective coefficient of 1000 W/(m^2·K) con be assumed (including radiation transfer). The outside is exposed to cooling water at 80 ºC, with which a convective coefficient of 5000 W/(m^2·K) con be assumed. The wall is made of cast iron 4 mm thick. Find:

a) The average state of the temperature profile within the wall.
b) The effect of the oscillations of gas temperature.

Se va a estudiar la transmisión de calor a través de la pared de un cilindro de un motor de combustión. El interior está expuesto a los gases de combustión, cuya temperatura media es de 1200 K con una oscilación de ±500 K (se trata de un pequeño motor de dos tiempos a 1800 rpm), con los cuales se estima que el coeficiente convectivo es de 1000 W/(m^2·K), incluyendo la radiación. El exterior está en contacto con el agua de refrigeración, que está a 80 ºC y cuyo coeficiente convectivo se estima en 5000 W/(m^2·K). El cilindro es de fundición de hierro de 4 mm de espesor. Se pide:

a) El perfil medio de temperatura en la pared del cilindro.
b) El efecto de la oscilación de la temperatura de los gases.

Solution

a) The average state of the temperature profile within the wall.

First, the geometry and nomenclature is sketched in Fig. 1, together with the expected solution. Notice that a planar geometry is assumed because the wall thickness is assumed much smaller than the diameter of the cylinder.

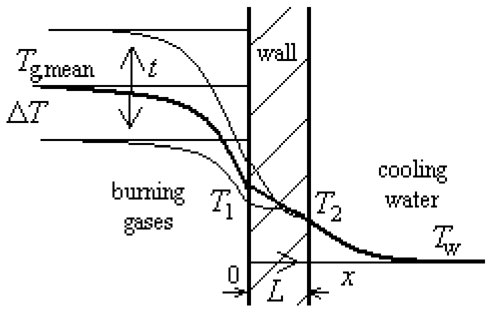

Fig. 1. Temperature profile across the chamber wall in a reciprocating-engine.

With this planar approximation, the average temperature profile within the wall, $0<x<L$, is the linear function $T(x)=T_1-(T_1-T_0)x/L$ (the exponential matching with the fluid temperatures far from the wall, sketched in Fig. 1, is just to illustrate the whole temperature field, but cannot be computed by traditional

heat-transfer means (it would require solving the whole fluid-dynamic problem with some CFD package). The end values, and the unitary heat flux, are:

$$\dot{q}=h_g\left(T_g-T_1\right)=k\frac{T_1-T_2}{L}=h_w\left(T_2-T_w\right)=\frac{T_g-T_w}{\frac{1}{h_g}+\frac{L}{k}+\frac{1}{h_w}}=\frac{1200-(80+273)}{\frac{1}{1000}+\frac{0.004}{50}+\frac{1}{5000}}=660\ \frac{\text{kW}}{\text{m}^2}$$

and T_1=538 K, and T_2=485 K; i.e. the wall adapts to mean values of 265 ºC inside and 212 ºC outside.

b) The effect of the oscillations of gas temperature.
This is a very complicated problem, even for numerical simulation, because of its transient periodic nature. The only simple way-out is to use the analytical solution of a periodic boundary-condition in a semi-infinite slab, hoping that the high-frequency excitation would give rise to a small penetration depth of the oscillations. Notice that we only look for the periodic solution, i.e. without regard of the initial transients after the oscillation starts.

If we try a solution to the heat equation within the solid wall, which is periodic in time with the applied period τ=1/30 s, and decays exponentially with a characteristic penetration length x_c to be found:

$$\frac{\partial^2 T}{\partial x^2}-\frac{1}{a}\frac{\partial T}{\partial t}=0\ \xrightarrow{T(x,t)=C\exp(-x/x_c)\cos(2\pi t/\tau-x/x_c)}\ 2C\frac{a\tau-\pi x_c^2}{a\tau x_c^2}\exp\left(-\frac{x}{x_c}\right)\sin\left(\frac{2\pi t}{\tau}-\frac{x}{x_c}\right)=0$$

$$\Rightarrow\ x_c=\sqrt{\frac{a\tau}{\pi}}\ \rightarrow\ \frac{T(x,t)-T_{0,\text{mean}}}{T_{0,\max}-T_{0,\text{mean}}}\overset{t>>\tau}{=}\exp\left(-\frac{x}{x_c}\right)\cos\left(\frac{2\pi t}{\tau}-\frac{x}{x_c}\right)$$

a being the thermal diffusivity of the solid. The variables $T_{0,\text{mean}}$ and $T_{0,\max}$ are the average and peak values of the temperature oscillation at the surface, which are not imposed in this convection-controlled exercise. Instead of the solid-surface temperature, the far-field temperature in a fluid in contact with that surface is imposed, $T_{fluid}(t)=T_{1,\text{mean}}+\Delta T_1\sin(2\pi t/\tau)$, but this can be solved introducing a damping in surface-temperature amplitude and a phase-shift in the time response, both to be computed from the convective heat-transfer coupling:

$$h\left(T_{fluid}(t)-T(0,t)\right)=-k\left.\frac{\partial T(x,t)}{\partial x}\right|_{x=0}$$

The solution to the temperature field within the solid, in terms of the data, is then:

$$\frac{T(x,t)-T_{1,mean}}{T_{1,\max}-T_{1,mean}}\overset{t>>\tau}{=}\frac{1}{\sqrt{1+\frac{2}{Bi}+\frac{2}{Bi^2}}}\exp\left(-\frac{x}{x_c}\right)\sin\left(\frac{2\pi t}{\tau}-\frac{x}{x_c}-\arctan\left(\frac{1}{1+Bi}\right)\right),\ Bi\equiv\frac{hx_c}{k}$$

with $T_{1,\text{mean}}$=1200 K the mean temperature of the combustion gases, $T_{1,\max}$=1200+500=1800 K is maximum value (i.e. the gas temperature oscillates from 800 K to 1800 K sinusoidally at 30 Hz).

The characteristic penetration depth is:

$$x_c = \sqrt{a\tau/\pi} = \sqrt{12.8\cdot(1/30)/\pi} = 0.37 \text{ mm}$$

and Biot number:

$$Bi = hx_c/k = 1000\cdot(0.37\cdot10^{-3})/50 = 0.0074$$

The value for the amplitude damping at the surface-temperature is $1\Big/\sqrt{1+\left(2/Bi\right)+\left(2/Bi^2\right)} = 0.0052$ (i.e., the surface temperature oscillates only ±500·0.0052=±2.6 K (to be superposed to the mean temperature value of 538 K, previously found), and the phase shift relative to the imposed gas oscillations $\arctan\left(1/\left(1+Bi\right)\right) = 0.78 \text{ rad}$ (i.e. 45° delay). Some temperature profiles are plot in Fig. 2.

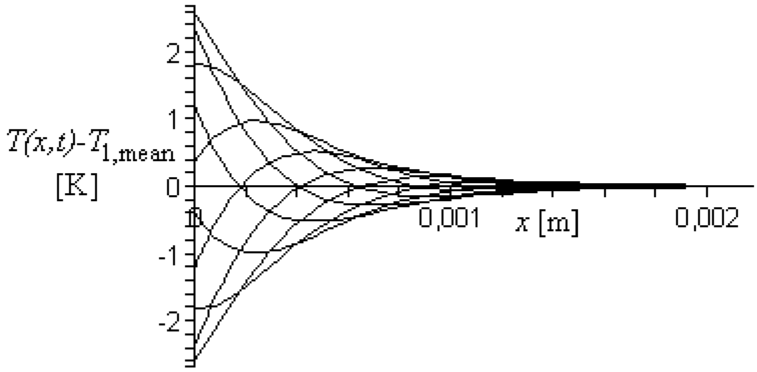

Fig. 2. Temperature oscillations in the wall (they are only noticeable near the inside surface).

In conclusion, the ±500 K oscillations in the burning gases only transmit as a ±2.6 K oscillations in surface temperature at the wall, with this amplitude decreasing further along the material, becoming unnoticeable at the outer end (notice that only half the width of the material, L=0.004 m, is shown in Fig. 2). Thus, the semi-infinite-slab model is appropriate.

Comments

One of the key advantages of reciprocating internal combustion engines is that cheap materials can be used in their construction (e.g. cast iron, against very expensive nickel alloys in gas turbines), because peak temperatures (up to 3000 K) are only realised within the burning gases, with an average temperature during the whole cycle of some 700 K, which would be the quasi-steady temperature level at the wall (because of its much larger thermal inertia), even without cooling; engine-wall cooling is applied to lower this 'adiabatic' temperature down to 600 K to prevent thermal decomposition of lubricating oil.

The small penetration of the temperature-oscillations explains why there are no problems of thermal fatigue in the wall material.

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