

ADDING WATER VAPOUR TO HUMID AIR

Statement

Ambient air at 25 °C, 93 kPa and 50%RH is forced to flow at a rate of 0.1 kg/s inside a duct, and a saturated (at 93 kPa) water-vapour flow-rate of 0.002 kg/s is added in a mixing chamber.

- a) Evaluate the dew and wet-bulb temperatures of the incoming air.
- b) Conditions after mixing.

A una corriente de 0,1 kg/s de aire atmosférico en condiciones 25 °C, 93 kPa y 50% de humedad relativa se le añade un flujo de vapor saturado a 93 kPa de 0,002 kg/s. Se pide:

- a) Temperaturas de rocío y de bulbo húmedo de la atmósfera.
- b) Condiciones tras el mezclado.

Solution

a) Evaluate the dew and wet-bulb temperatures of the incoming air. For the dew point temperature, $p^*(T_{dew}) = \phi p^*(T)$, using Antoine equation, one gets an explicit form:

$$\ln \frac{p^{*}(T_{\text{dew}})}{p_{0}} = \ln \phi + \ln \frac{p^{*}(T)}{p_{0}} \rightarrow A - \frac{B}{\frac{T_{\text{dew}}}{T_{0}} + C} = \ln \phi + A - \frac{B}{\frac{T}{T_{0}} + C}$$
$$\rightarrow \frac{T_{\text{dew}}}{T_{0}} = \frac{1}{\frac{1}{\frac{1}{T_{0}} + C} - \frac{\ln \phi}{B}} - C = \frac{1}{\frac{1}{\frac{298 \text{ K}}{1 \text{ K}} + (-39)} - \frac{\ln 0.5}{3985}} - (-39) \rightarrow T_{\text{dew}} = 13.8 \text{ }^{\circ}\text{C}$$

but for the wet-bulb temperatures we have an implicit equation:

$$c_{pa} \left(T_{1} - T_{0}\right) + \frac{M_{va}}{\frac{p}{\phi_{1} p^{*}(T_{1})} - 1} h_{LV0} = c_{pa} \left(T_{dew} - T_{0}\right) + \frac{M_{va}}{\frac{p}{p^{*}(T_{dew})} - 1} h_{LV0}$$

$$(1000 \text{ J/(kg \cdot K)}) \left(298 \text{ K} - 273 \text{ K}\right) + \frac{0.622}{\frac{93 \text{ kPa}}{0.5 \cdot (3.17 \text{ kPa})} - 1} \left(2.5 \text{ MJ/kg}\right) =$$

$$= (1000 \text{ J/(kg \cdot K)}) \left(T_{dew} - 273 \text{ K}\right) + \frac{0.622}{\frac{93 \text{ kPa}}{p^{*}(T_{dew})} - 1} \left(2.5 \text{ MJ/kg}\right)$$

solved by iterations (a good start is simply the actual temperature) to yield $T_{wet}=17.4$ °C.

b) Conditions after mixing.

If we assume that the exit is still unsaturated humid air, the equations are:

mass balance for dry air: $\dot{m}_{a1} = \dot{m}_{a2}$ mass balance for water: $\dot{m}_{a1}w_1 + \dot{m}_3 = \dot{m}_{a2}w_2$ energy balance: $\dot{m}_{a1}h_1 + \dot{m}_3h_3 = \dot{m}_{a2}h_2$

that reduce to one equation with one unknown, T_2 :

$$c_{pa}\left(T_{1}-T_{0}\right)+w_{1}h_{LV0}+\frac{\dot{m}_{3}}{\dot{m}}h_{3}=c_{pa}\left(T_{2}-T_{0}\right)+\left(w_{1}+\frac{\dot{m}_{3}}{\dot{m}}\right)h_{LV0}$$

the other variables being known:

$$w_{1} = \frac{0.622}{\frac{93 \text{ kPa}}{0.5 \cdot (3.17 \text{ kPa})} - 1} = 0.011$$

$$h_{1} = c_{\text{pa}}(T_{1} - T_{0}) + w_{1}h_{\text{LV0}} = 1000 \cdot (298 - 273) + 0.011 \cdot 2.5 \cdot 10^{6} = 52.5 \text{ kJ/kg}$$

$$h_{3} = c_{\text{L}}(T_{\text{b}} - T_{0}) + h_{\text{LVb}} + c_{\text{pv}}(T_{3} - T_{\text{b}}) = 4200 \cdot (371 - 273) + 2.26 \cdot 10^{6} + 2000 \cdot (371 - 373) = 2670 \text{ kJ/kg}$$

$$\frac{\dot{m}_{3}}{\dot{m}} = \frac{0.002}{0.1} = 0.02$$

and so on, yielding as result $T_2=301$ K. But if we now compute the relative humidity for these values (p,T,w) we find $\phi>1$ (actually 1.2), what is incompatible with the assumption of unsaturated exit and we have to rework the mixing problem.

Now we know that the exit is a two-phase flow (a mist), and naming \dot{m}_4 the condensate massflow-rate, the balance equations are:

mass balance for dry air: $\dot{m}_{a1} = \dot{m}_{a2}$ mass balance for water: $\dot{m}_{a1}w_1 + \dot{m}_3 = \dot{m}_{a2}w_2 + \dot{m}_4$ energy balance: $\dot{m}_{a1}h_1 + \dot{m}_3h_3 = \dot{m}_{a2}h_2 + \dot{m}_4h_4$

where now we know that $\phi_3=1$ instead of $w_1=0.011+0.02=0.013$. Thus, we get a two-equations system in two-unknowns (T_2, \dot{m}_4) that when solved yields $T_2=303$ K and $\dot{m}_4=0.1\cdot10^{-3}$ kg/s.

Comments. Of course, if we had started assuming a mist at the exit, some work would have been spared.

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