ADDING WATER VAPOUR TO HUMID AIR

Statement

Ambient air at 25 ºC, 93 kPa and 50%RH is forced to flow at a rate of 0.1 kg/s inside a duct, and a saturated (at 93 kPa) water-vapour flow-rate of 0.002 kg/s is added in a mixing chamber.

a) Evaluate the dew and wet-bulb temperatures of the incoming air.

b) Conditions after mixing.

A una corriente de 0,1 kg/s de aire atmosférico en condiciones 25 ºC, 93 kPa y 50% de humedad relativa se le añade un flujo de vapor saturado a 93 kPa de 0,002 kg/s. Se pide:

a) Temperaturas de rocío y de bulbo húmedo de la atmósfera.

b) Condiciones tras el mezclado.

Solution

a) Evaluate the dew and wet-bulb temperatures of the incoming air.

For the dew point temperature, $p^*(T_{\text{dew}})=\phi\, p^*(T)$, using Antoine equation, one gets an explicit form:

$$\ln\frac{p^*(T_{\text{dew}})}{p_0}=\ln\phi+\ln\frac{p^*(T)}{p_0}\quad\to\quad A-\frac{B}{\frac{T_{\text{dew}}}{T_0}+C}=\ln\phi+A-\frac{B}{\frac{T}{T_0}+C}$$

$$\to\quad\frac{T_{\text{dew}}}{T_0}=\frac{1}{\frac{1}{\frac{T}{T_0}+C}-\frac{\ln\phi}{B}}-C=\frac{1}{\frac{1}{\frac{298\text{ K}}{1\text{ K}}+(-39)}-\frac{\ln 0.5}{3985}}-(-39)\quad\to\quad T_{\text{dew}}=13.8\text{ ºC}$$

but for the wet-bulb temperatures we have an implicit equation:

$$c_{pa}\left(T_1-T_0\right)+\frac{M_{va}}{\frac{p}{\phi_1\, p^*(T_1)}-1}h_{LV0}=c_{pa}\left(T_{dew}-T_0\right)+\frac{M_{va}}{\frac{p}{p^*(T_{dew})}-1}h_{LV0}$$

$$(1000\text{ J/(kg}\cdot\text{K))}\left(298\text{ K - }273\text{ K}\right)+\frac{0.622}{\frac{93\text{ kPa}}{0.5\cdot(3.17\text{ kPa})}-1}\left(2.5\text{ MJ/kg}\right)=$$

$$=(1000\text{ J/(kg}\cdot\text{K))}\left(T_{dew}-273\text{ K}\right)+\frac{0.622}{\frac{93\text{ kPa}}{p^*(T_{dew})}-1}\left(2.5\text{ MJ/kg}\right)$$

solved by iterations (a good start is simply the actual temperature) to yield T_{wet}=17.4 ºC.

b) Conditions after mixing.

If we assume that the exit is still unsaturated humid air, the equations are:

mass balance for dry air: $\dot{m}_{a1} = \dot{m}_{a2}$
mass balance for water: $\dot{m}_{a1}w_1 + \dot{m}_3 = \dot{m}_{a2}w_2$
energy balance: $\dot{m}_{a1}h_1 + \dot{m}_3h_3 = \dot{m}_{a2}h_2$

that reduce to one equation with one unknown, T_2:

$$c_{pa}\left(T_1 - T_0\right) + w_1 h_{LV0} + \frac{\dot{m}_3}{\dot{m}}h_3 = c_{pa}\left(T_2 - T_0\right) + \left(w_1 + \frac{\dot{m}_3}{\dot{m}}\right)h_{LV0}$$

the other variables being known:

$$w_1 = \frac{0.622}{\dfrac{93\text{ kPa}}{0.5\cdot(3.17\text{ kPa})} - 1} = 0.011$$

$h_1=c_{pa}(T_1-T_0)+w_1h_{LV0}=1000\cdot(298-273)+0.011\cdot2.5\cdot10^6=52.5$ kJ/kg

$h_3=c_L(T_b-T_0)+h_{LVb}+c_{pv}(T_3-T_b)=4200\cdot(371-273)+2.26\cdot10^6+2000\cdot(371-373)=2670$ kJ/kg

$$\frac{\dot{m}_3}{\dot{m}} = \frac{0.002}{0.1} = 0.02$$

and so on, yielding as result T_2=301 K. But if we now compute the relative humidity for these values (p,T,w) we find ϕ>1 (actually 1.2), what is incompatible with the assumption of unsaturated exit and we have to rework the mixing problem.

Now we know that the exit is a two-phase flow (a mist), and naming $\dot{m}_4$ the condensate mass-flow-rate, the balance equations are:

mass balance for dry air: $\dot{m}_{a1} = \dot{m}_{a2}$
mass balance for water: $\dot{m}_{a1}w_1 + \dot{m}_3 = \dot{m}_{a2}w_2 + \dot{m}_4$
energy balance: $\dot{m}_{a1}h_1 + \dot{m}_3h_3 = \dot{m}_{a2}h_2 + \dot{m}_4h_4$

where now we know that ϕ_3=1 instead of w_1=0.011+0.02=0.013. Thus, we get a two-equations system in two-unknowns ($T_2, \dot{m}_4$) that when solved yields T_2=303 K and $\dot{m}_4$=$0.1\cdot10^{-3}$ kg/s.

Comments. Of course, if we had started assuming a mist at the exit, some work would have been spared.

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