

## ADDING WATER VAPOUR TO HUMID AIR

### Statement

Ambient air at 25 °C, 93 kPa and 50%RH is forced to flow at a rate of 0.1 kg/s inside a duct, and a saturated (at 93 kPa) water-vapour flow-rate of 0.001 kg/s is added in a mixing chamber.

- Evaluate the dew and wet-bulb temperatures of the incoming air.
- Conditions after mixing.

🇪🇸 A una corriente de 0,1 kg/s de aire atmosférico en condiciones 25 °C, 93 kPa y 50% de humedad relativa se le añade un flujo de vapor saturado a 93 kPa de 0,001 kg/s. Se pide:

- Temperaturas de rocío y de bulbo húmedo de la atmósfera.
- Condiciones tras el mezclado.

### Solution

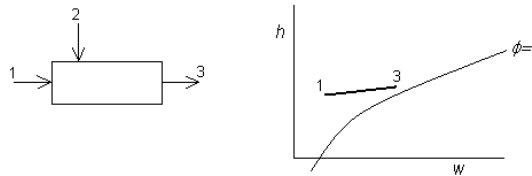


Fig. 1. Sketch of the process.

- Evaluate the dew and wet-bulb temperatures of the incoming air.

For the dew point temperature,  $p^*(T_{\text{dew}}) = \phi p^*(T)$ , using Antoine equation, one gets an explicit form:

$$\ln \frac{p^*(T_{\text{dew}})}{p_0} = \ln \phi + \ln \frac{p^*(T)}{p_0} \rightarrow A - \frac{B}{\frac{T_{\text{dew}}}{T_0} + C} = \ln \phi + A - \frac{B}{\frac{T}{T_0} + C}$$

$$\rightarrow \frac{T_{\text{dew}}}{T_0} = \frac{1}{\frac{1}{\frac{T}{T_0} + C} - \frac{\ln \phi}{B}} - C = \frac{1}{\frac{1}{\frac{298 \text{ K}}{1 \text{ K}} + (-39)} - \frac{\ln 0.5}{3985}} - (-39) \rightarrow T_{\text{dew}} = 13.8 \text{ °C}$$

but for the wet-bulb temperatures we have an implicit equation:

$$c_{pa}(T_1 - T_0) + \frac{M_{va}}{\frac{p}{\phi_1 p^*(T_1)} - 1} h_{LV_0} = c_{pa}(T_{\text{wet}} - T_0) + \frac{M_{va}}{\frac{p}{p^*(T_{\text{wet}})} - 1} h_{LV_0}$$

$$(1000 \text{ J/(kg} \cdot \text{K)})(298 \text{ K} - 273 \text{ K}) + \frac{0.622}{0.5 \cdot (3.17 \text{ kPa})} (2.5 \text{ MJ/kg}) =$$

$$= (1000 \text{ J/(kg} \cdot \text{K)})(T_{\text{wet}} - 273 \text{ K}) + \frac{0.622}{\frac{93 \text{ kPa}}{p^*(T_{\text{wet}})} - 1} (2.5 \text{ MJ/kg})$$

which must be solved by iterations (a good start is the actual temperature), or simply by linear-root-finding between the residuals at the actual temperature and at the dew-point temperature), yielding  $T_{\text{wet}}=17.4$  °C. Warning: mind that the iterative procedure can diverge if improperly followed (e.g. it diverges if the linear term in  $T_{\text{wet}}$  is solved for, a value is given to  $T_{\text{wet}}$  in the other term, and the resulting  $T_{\text{wet}}$  iterated).

b) Conditions after mixing.

If we assume that the exit is still unsaturated humid air, the equations are:

$$\text{mass balance for dry air: } \dot{m}_{a1} = \dot{m}_{a3}$$

$$\text{mass balance for water: } \dot{m}_{a1}w_1 + \dot{m}_2 = \dot{m}_{a3}w_3$$

$$\text{energy balance: } \dot{m}_{a1}h_1 + \dot{m}_2h_2 = \dot{m}_{a3}h_3$$

that reduce to one equation with one unknown,  $T_3$ :

$$c_{pa}(T_1 - T_0) + w_1 h_{LV0} + \frac{\dot{m}_2}{\dot{m}} h_2 = c_{pa}(T_3 - T_0) + \left( w_1 + \frac{\dot{m}_2}{\dot{m}} \right) h_{LV0}$$

the other variables, absolute humidity, air enthalpy, and vapour enthalpy, at the entrance, being known:

$$w_1 = \frac{0.622}{\frac{93 \text{ kPa}}{0.5 \cdot (3.17 \text{ kPa})} - 1} = 0.011$$

$$h_1 = c_{pa}(T_1 - T_0) + w_1 h_{LV0} = 1000 \cdot (298 - 273) + 0.011 \cdot 2.5 \cdot 10^6 = 52.5 \text{ kJ/kg}$$

$$h_2 = c_L(T_b - T_0) + h_{LVb} + c_{pv}(T_2 - T_b) = 4200 \cdot (373 - 273) + 2.26 \cdot 10^6 + 2000 \cdot (371 - 373) = 2670 \text{ kJ/kg}$$

$$\frac{\dot{m}_2}{\dot{m}} = \frac{0.001}{0.1} = 0.01$$

and so on, yielding as result  $T_3=299$  K (26 °C). Now we compute the relative humidity for these values ( $p, T, w$ ) and we find  $\phi_3=0.88$ .

**Comments.** Notice that the assumption of unsaturated exit must be checked by computing the relative humidity and checking that  $\phi < 1$ . If more water vapour were added, the exit could be super-saturated (see [Ex. 8.4: Adding water vapour to humid air](#)).

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