EFFECT OF THERMAL EXPANSION ON A PENDULUM

Statement

Find the sensitivity of a pendulum-clock with temperature, assuming a stainless-steel rod that beats with a 2 s period at 20 ºC, and find the discrepancy after a day at 30 ºC

Determinar el efecto de la dilatación térmica sobre un reloj de péndulo, suponiendo que es de acero inoxidable, que tiene un periodo de 2 s a 20 ºC, y que está un día a 30 ºC.

Solution.

From Mechanics, the period of a pendulum and the corresponding length are:

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \rightarrow \quad L = g\left(\frac{T}{2\pi}\right)^2 = 9.8\left(\frac{2}{2\pi}\right)^2 = 0.993\ \mathrm{m}$$

With a typical thermal expansion for stainless-steel of α=14·10^{-6} 1/K (Table ../eSol.htm), the elongation is $\Delta L=L\alpha\Delta T$=0.14 mm and the sensitivity:

$$\frac{\mathrm{d}\tau}{\tau} = \frac{1}{2}\frac{\mathrm{d}L}{L} = \frac{\alpha\mathrm{d}T}{2} \quad \rightarrow \quad \frac{\Delta\tau}{\tau} = \frac{14\cdot10^{-6}\cdot10}{2} = 70\cdot10^{-6}$$

that for 86400 s means a lag of 86300·70·10^{-6}=6 s, i.e. the clock lags behind real time.

Comments.

Notice that the sensitivity, and thus the discrepancy, is independent of the rod length.

During the French Revolution, when a new standard of length was been discussed, it seems that Talleyrand defended the idea of using the length of a pendulum that beats seconds, instead of the finally adopted 1/10^7 part of the meridian quadrant.

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