SOUND SPEED IN AIR AND WATER

Statement

Deduce the sound speed in air and water, from the general equation $c=\sqrt{\partial p/\partial\rho|_s}$

Deducir expresiones particulares para la velocidad del sonido en el aire y en el agua a partir de: $c=\sqrt{\partial p/\partial\rho|_s}$

Solution.

For air, assuming a perfect gas, isentropic means $pv^\gamma=p\rho^{-\gamma}$=constant, and thus $\frac{dp}{p}-\gamma\frac{d\rho}{\rho}=0$, and consequently $c=\sqrt{\gamma\frac{p}{\rho}}=\sqrt{\gamma RT}$, with a typical value of $\sqrt{\gamma RT}=\sqrt{1.4\cdot 287\cdot 288}$ =340 m/s at 15 ºC,

For water, isentropic is equivalent to isotherm as a first approximation (the perfect liquid approximation), and from $\kappa\equiv\frac{-1}{V}\frac{\partial V}{\partial p}\Big|_T=\frac{1}{\rho}\frac{\partial\rho}{\partial p}\Big|_T$ one deduces $c=\sqrt{\frac{1}{\rho\kappa}}$, with a typical value of $\sqrt{\frac{1}{\rho\kappa}}=\sqrt{\frac{1}{1000\cdot(0.45\cdot 10^{-9})}}$ =1490 m/s.

Comments. Notice that the sound speed in air grows with T but does nor change with p (within the perfect gas approximation), whereas for water, the speed of sound would be infinite with the incompressible model.

More accurate values for water at 100 kPa are:

	Density ρ [kg/m^3]	Thermal expansion α [K^{-1}]	Compressibility κ [Pa^{-1}]	Sound speed c [m/s]	Thermal capacity c_p [J/(kg·K)]
Pure water at 0 ºC	999.84	$-68\cdot10^{-6}$	$0.51 10^{-9}$	1402	4218
at 15 ºC	999.10	$152\cdot10^{-6}$	$0.45 10^{-9}$	1509	4187
at 30 ºC	995.65	$303\cdot10^{-6}$	$0.45 10^{-9}$	1509	4178
at 50 ºC	992.22	$455\cdot10^{-6}$	$0.44 10^{-9}$	1543	4181
at 70 ºC	977.78	$580\cdot10^{-6}$	$0.45 10^{-9}$	1555	4189
at 100 ºC	958.40	$750\cdot10^{-6}$	$0.49 10^{-9}$	1543	4216
Seawater at 0 ºC	1028	$53\cdot10^{-6}$	$0.463 10^{-9}$	1449	3986
at 15 ºC	1026	$212\cdot10^{-6}$	$0.430 10^{-9}$	1507	3990
at 30 ºC	1023	$334\cdot10^{-6}$	$0.410 10^{-9}$	1546	4001
Seawater 0 ºC, 100 MPa	1071	$262\cdot10^{-6}$	$0.354 10^{-9}$	1623	3769

Ref.: http://fermi.jhuapl.edu/denscalc.html.

Notice that with this data you may compute $c_v=c_p-\alpha^2T/(\kappa\rho)$; e.g. for pure water at 0 ºC c_p-c_v=2.5 J/(kg·K).

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