MAXIMUM WORK FROM TWO BODIES

Statement

Find the maximum work obtainable from two solid bodies, of given thermal capacities and initial temperatures T_1 y T_2, in absence of a surrounding atmosphere, with application to equal thermal capacities.

Calcular el trabajo máximo obtenible de dos cuerpos sólidos de masas y capacidad térmicas dadas, inicialmente a temperatura T_1 y T_2, en ausencia de atmósfera circundante, con aplicación al caso de capacidades térmicas iguales.

Solution. In algebraic terms, the work produced being negative, its maximum value is the algebraic minimum, known to happen when no entropy is produced in the process:

$$W_{\min} = W\big|_{\Delta S_{\text{univ}}=0}$$

Since the two-stone system is isolated, Q=0 (but W≠0 because we want to extract work); thence, assuming perfect calorific substances:

$$W = \Delta E = m_1 c_1 \left(T_{eq} - T_1\right) + m_2 c_2 \left(T_{eq} - T_2\right)$$

where the final equilibrium temperature is to be found subject to the constrain:

$$\Delta S_{\text{univ}} = m_1 c_1 \ln\frac{T_{eq}}{T_1} + m_2 c_2 \frac{T_{eq}}{T_2}$$

what gives the solution, that has a simple form in the case of equal thermal capacities:

If $m_1c_1=m_2c_2$ then $T_{eq} = \sqrt{T_1 T_2}$ and $W_{\min} = mc(2\sqrt{T_1 T_2} - T_1 - T_2)$.

Notice that if the two bodies reach equilibrium by thermal contact, no work would be obtained, and the final temperature would be the arithmetic mean (for equal capacities), with irreversibility (work lost) of:

$$I \equiv W_u - W_{\text{u}}\big|_{\Delta S_{\text{univ}}=0} = 0 - W_{\min}$$

Comments. Firstly, it must be understood, in this and any other statement of exergy analysis, that the universe described determine the consumable systems, but any other non-consumable device may be needed. Additionally, notice that by separate, a hot body and a cold body have no exergy, but they do have as an ensemble.

We use $W_{\min}$ for the algebraic value (maximum of a negative variable since energy is extracted from the combined system.

Notice that the equilibrium temperature is the geometric mean (always lower than the arithmetic mean.

As for the numerical values in this imaginary problem, from two stones with m=1000 kg, c=1000 J/(kg·K), T_1=300 K and T_2=400 K, only 7 MJ might be obtained (the equivalent to 0.2 kg of a common fuel, and much more cumbersome in practice).

As for the actual device to be used to extract in practice that work, a small heat engine of a closed circuit type (e.g. using butane as working fluid in a vapour cycle, to be explained in Chapter 17) could produce a sizeable part of it.

A final comment is that we do not live in a two-stone world, and hence it is difficult to think of a suitable practical example. The closest example might be a geothermal source (a large underground hot-rock and the temperate environment we live in).

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