U-TUBE USED TO MEASURE TEMPERATURE AND PRESSURE

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Statement

A U-tube is made by joining two 1 m vertical glass-tubes of 3 mm bore (6 mm external diameter) with a short tube at the bottom. Water is poured until the liquid fills 600 mm in each column. Then, one end is closed. Find:

a) The change in menisci height due to an ambient pressure change, $(\partial z/\partial p_{amb})$, with application to Δp=1 kPa.

b) The change in menisci height due to an ambient temperature change, $(\partial z/\partial T_{amb})$, with application to ΔT=5 ºC.

Un manómetro en U, de vidrio de 3 mm de diámetro interior y 6 mm de diámetro exterior, tiene los lados de 1 m de altura y la conexión por abajo de longitud despreciable. El tubo se ha llenado con agua hasta 600 mm de altura en cada lado, y se ha tapado una de las bocas. Se pide:

a) Determinar el efecto del cambio de presión ambiente sobre las alturas de los meniscos $(\partial z/\partial p_{amb})$, particularizando para Δp=1 kPa.

b) Determinar el efecto del cambio de temperatura ambiente sobre las alturas de los meniscos $(\partial z/\partial T_{amb})$, particularizando para ΔT=5 ºC.

Solution

The key to this problem is the closure at one end, what creates a separate trapped-gas system, responsive to environmental changes (otherwise, any pressure or temperature change would be unnoticeable with both ends open). It is intuitive that an increase in ambient temperature will force the trapped gas to expand, and an increase in ambient pressure will compress the trapped gas, as sketched in Fig. 1, where it is also pointed out that we take as reference for heights the bottom of the U-tube.

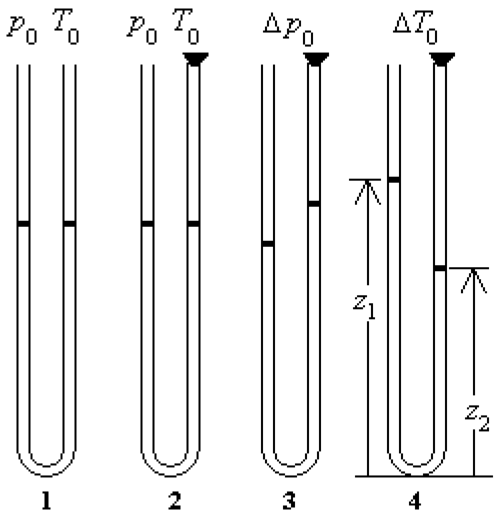

Fig. 1. U-tube partially filled with water, closure at one end, and effect of pressure and temperature changes.

Some typical assumptions concerning air-water equilibrium are adopted: the effects of air dissolving in water, and of water dissolving in air (much more important), are neglected. Constant density is assumed for both solids and liquids, and ideal-gas behaviour for the trapped gas, $pV=mRT$. Hydrostatic pressure effects are retained in the liquid, but neglected in the gas.

Initial conditions after closing one end (state 2 in Fig. 1; do not confuse temporal states (1, 2, 3, and 4), with spatial states (L=left, open column, R=right, closed column):

- Left column: z_{2L}=0,6 m, $p_{2L}=p_0=10^5$ Pa, $T_{2L}=T_0$=288 K (15 ºC).
- Right column: z_{2R}=0,6 m, $p_{2R}=p_0=10^5$ Pa, $T_{2R}=T_0$=288 K (15 ºC). Mass of air trapped, m:

$$m=\frac{pV}{RT}=\frac{p_{2R}A\left(L-z_{2R}\right)}{RT_0}=\frac{10^5\cdot 7.1^{-6}\left(1-0.6\right)}{287\cdot 288}=3.4^{-6}\text{ kg}$$

where the cross-section area $A=\pi D^2/4=\pi(0.003)^2/4=7.1\cdot 10^{-6}$ m^2 and air constant $R=R_u/M$=8.3/0.029=287 J/(kg·K), have been substituted. Of course, the precise amount of air trapped is of secondary importance, the key point is that it does not change with environmental changes. Another key point is that, at equilibrium, pressure at the bottom of the U-tube must be the same as seen from the left or from the right: $p_0+\rho g z_{2L}=p_0+\rho g z_{2R}$, where ρ is the liquid density, and all pressure variables refer to the value at the corresponding gas-liquid interface.

a) The change in menisci height due to an ambient pressure change, $(\partial z/\partial p_{amb})$, with application to Δp=1 kPa.

Conditions after a barometric pressure change Δp_0 at constant temperature (state 3 in Fig. 1):

- Left column: z_L decreases an unknown amount, $p_{3L}=p_0+\Delta p_0$, $T_{3L}=T_0$=288 K.
- Right column: z_R increases, but z_L+z_R=1.2 m remain constant (0.6 m + 0.6 m, initially) by liquid-mass conservation and constant-density model). p_{3R} is unknown, but $T_{3R}=T_0$=288 K. Mass conservation of the trapped air, of liquid mass, and pressure balance at the bottom give:

$$\left.\begin{array}{l} m=\dfrac{pV}{RT}=\dfrac{p_{3R}A_0\left(L-z_{3R}\right)}{RT_0} \\ z_{3L}+z_{3R}=2z_0 \\ p_{3L}+\rho g z_{3L}=p_{3R}+\rho g z_{3R} \end{array}\right\}$$

Although it reduces to one second order algebraic equation, the system of equations would be much easier to solve if we only take variations in the magnitudes, and make the linearization of retaining only first-order terms, since the relative excitation $\Delta p_0/p_0$<<1 allows to neglect second-order effects:

$$\left.\begin{array}{l} pV=mRT\longrightarrow\dfrac{\mathrm{d}p}{p}+\dfrac{\mathrm{d}V}{V}=\dfrac{\mathrm{d}T}{T}\longrightarrow\dfrac{\mathrm{d}p_{3R}}{p_0}+\dfrac{A\mathrm{d}\left(L-z_{3R}\right)}{A\left(L-z_0\right)}=0 \\ z_{3L}+z_{3R}=2z_0\longrightarrow\mathrm{d}z_{3L}+\mathrm{d}z_{3R}=0 \\ p_{3L}+\rho g z_{3L}=p_{3R}+\rho g z_{3R}\longrightarrow\mathrm{d}p_0+\rho g\mathrm{d}z_{3L}=\mathrm{d}p_{3R}+\rho g\mathrm{d}z_{3R} \end{array}\right\}\Rightarrow\frac{\mathrm{d}z_{3R}}{\mathrm{d}p_0}=\frac{1}{2\rho g+\dfrac{p_0}{L-z_0}}$$

which, upon substitution of the particular data, yields $\mathrm{d}z_{3R}/\mathrm{d}p_0$=1/(2·1000·9.8+$10^5$/(1–0.6))=40·$10^{-9}$ m/Pa (4 mm/kPa); i.e., the device senses barometric-pressure changes with a sensitivity coefficient of 4 mm/kPa, so that for a 1 kPa pressure rise (typical of going into good weather), the right meniscus rises 4 mm and the left one descends 4 mm).

b) The change in menisci height due to an ambient temperature change, ($\partial z/\partial T_{amb}$), with application to ΔT=5 ºC.

Similarly, for the conditions after an overall temperature change ΔT_0 at constant pressure (state 4 in Fig. 1):

$$\left.\begin{array}{l} pV = mRT \longrightarrow \dfrac{\mathrm{d}p}{p} + \dfrac{\mathrm{d}V}{V} = \dfrac{\mathrm{d}T}{T} \longrightarrow \dfrac{\mathrm{d}p_{4R}}{p_0} + \dfrac{A\mathrm{d}\left(L - z_{4R}\right)}{A\left(L - z_0\right)} = \dfrac{\mathrm{d}T_0}{T_0} \\ z_{4L} + z_{4R} = 2z_0 \longrightarrow \mathrm{d}z_{4L} + \mathrm{d}z_{4R} = 0 \\ p_{4L} + \rho g z_{4L} = p_{4R} + \rho g z_{4R} \longrightarrow \rho g \mathrm{d}z_{4L} = \mathrm{d}p_{4R} + \rho g \mathrm{d}z_{4R} \end{array}\right\} \Rightarrow \frac{\mathrm{d}z_{4R}}{\mathrm{d}T_0} = \frac{-p_0}{T_0\left(2\rho g + \dfrac{p0}{L - z_0}\right)}$$

which, upon substitution of the particular data, yields $\mathrm{d}z_{3R}/\mathrm{d}T_0$=–($10^5$/288)/(2·1000·9.8+$10^5$/(1–0.6))= –1.2·$10^{-3}$ m/K (1.2 mm/ºC)); i.e., the device senses ambient-temperature changes with a sensitivity coefficient of 1.2 mm/K, so that for a 5 ºC temperature rise (typical of in outdoor air during a few-hours interval during day time), the right meniscus descends 6 mm and the left one ascends 6 mm).

Comments

The U-tube arrangement is a standard manometric device (see Piezometry), but rarely used as a thermometer because there are other devices more handy for the latter purpose (see Thermometry).

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