



HEAT EXCHANGERS

Heat exchangers: types.....	1
Plate heat exchangers	4
Shell-and-tube heat exchangers	5
Boilers	6
Characteristics.....	8
Heat exchanger analysis.....	10
Overall heat-transfer coefficient	10
Fouling	11
Heat exchanger modelling	12
Non-dimensional method of heat-exchanger modelling: the NTU approach.....	15
Preliminary sizing	17

HEAT EXCHANGERS: TYPES

Heat exchangers are off-the-shelf equipment targeted to the efficient transfer of heat from a hot fluid flow to a cold fluid flow, in most cases through an intermediate metallic wall and without moving parts. We here focus on the thermal analysis of heat exchangers, but proper design and use requires additional fluid-dynamic analysis (for each fluid flow), mechanical analysis (for closure and resistance), materials compatibility, and so on.

Heat losses or gains of a whole heat exchanger with the environment can be neglected in comparison with the heat flow between both fluid flows; i.e. a heat exchanger can be assumed globally adiabatic. Thermal inertia of a heat exchanger is often negligible too (except in special cases when a massive porous solid is used as intermediate medium), and steady state can be assumed, reducing the generic energy balance to:

$$\Delta E = W + Q + \sum_{\text{time}}^{\text{openings}} \int h_{t,e} dm_e \rightarrow 0 = \dot{m}_1 \Delta h_1 + \dot{m}_2 \Delta h_2 \quad (1)$$

where the total enthalpy h_t has been approximated by enthalpy (i.e. negligible mechanical energy against thermal energy), and Δ means output minus input.

Although heat flows from hot fluid to cold fluid by thermal conduction through the separating wall (except in direct-contact types), heat exchangers are basically heat convection equipment, since it is the convective transfer what governs its performance. Convection within a heat exchanger is always forced, and may be with or without phase change of one or both fluids.

When one just relies in natural convection to the environment, like in the space-heating hot-water home radiator, or the domestic fridge back-radiator, they are termed 'radiators' (in spite of convection being dominant), and not heat exchangers. When a fan is used to force the flow of ambient air (or when natural

or artificial wind applies, like for car radiator) the name heat exchanger is often reserved for the case where the ambient fluid is ducted. Other names are used for special cases, like 'condenser' for the case when one fluid flow changes from vapour to liquid, 'vaporiser' (or evaporator, or boiler) when a fluid changes from liquid to vapour, or the 'cooling tower' dealt with below.

Devices with just one fluid flow (like a solar collector, a spacecraft radiator, a submerged electrical heater, or a simple pipe with heat exchange with the environment) are never named heat exchangers.

The basic designs for heat exchangers are the shell-and-tube heat exchanger and the plate heat exchanger, although many other configurations have been developed. According to flow layout, heat exchangers are grouped in:

- Shell-and-tube heat exchanger (STHE), where one flow goes along a bunch of tubes and the other within an outer shell, parallel to the tubes, or in cross-flow (Fig. 1a shows a typical example of STHE; details presented below).
- Plate heat exchanger (PHE), where corrugated plates are held in contact and the two fluids flow separately along adjacent channels in the corrugation (Fig. 1b shows details of the interior of a PHE; more details are presented below).
- Open-flow heat exchanger, where one of the flows is not confined within the equipment (or at least, like in Fig. 1c, not specifically piped). They originate from air-cooled tube-banks, and are mainly used for final heat release from a liquid to ambient air, as in the car radiator, but also used in vaporisers and condensers in air-conditioning and refrigeration applications, and in directly-fired home water heaters. When gases flow along both sides, the overall heat-transfer coefficient is very poor, and the best solution is to make use of heat-pipes as intermediate heat-transfer devices between the gas streams; otherwise, finned separating surfaces, or, better, direct contact through a solid heat-regenerator (also named heat-recuperator, although sometimes the last term is restricted to fluid-to-fluid heat recovery with a conventional heat exchanger), are used.
- Contact heat exchanger, where the two fluids enter into direct contact (simultaneous heat and mass transfer takes place). Furthermore, the contact can be continuous, i.e. when the two fluids mix together and then separate by gravity forces, as in a [cooling tower](#), or the contact can be alternatively with a third medium, usually solid, as in regenerative heat exchangers (RHE), like the rotating wheel shown in Fig. 1d (the hot gas heats the wheel whereas the cold gas retrieves that energy). When the heat-exchange process between the hot and the cold fluids is delayed significantly, the term 'thermal energy storage' is used instead of RGE. There is always some contamination by entrainment of one fluid by the other, although many times it is irrelevant (as in air-conditioning heat-regenerators), or even intended (as in cooling towers). Notice also that, if the mixed-up fluids do not separate, as in open feed-water heaters or in evaporative coolers, the device is not named heat exchanger but just heater or cooler.

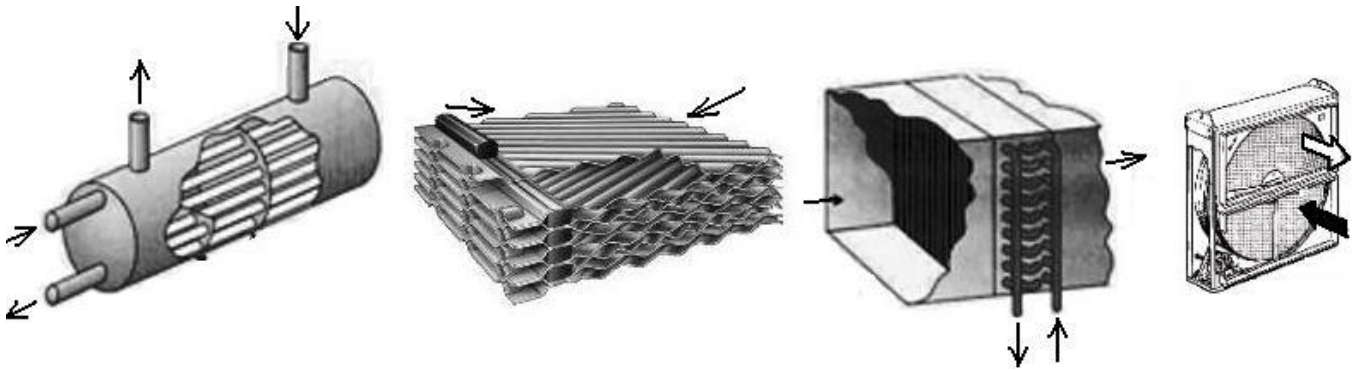


Fig. 1. Types of heat exchangers: a) shell-and-tube, b) plates, c) open-flow, d) rotating-wheel.

Additionally, heat exchangers may be classified according to the type of fluid used (liquid-to-liquid, liquid-to-gas, gas-to-liquid, gas-to-gas), according to phase changes (vaporisers, condensers), according to relative flow direction (counter-flow, co-flow, cross-flow), according to area density (transfer area per unit volume) or channel size, etc. In terms of the smallest hydraulic diameter of the two flows, D_h , or the area density, β (typical correlation is $\beta \approx 3/D_h$), heat exchangers may be grouped as:

- Conventional or non-compact heat exchangers, if $D_h > 5$ mm, or $\beta < 400$ m²/m³.
- Compact heat exchangers, if $1 < D_h/\text{mm} < 5$, or 400 m²/m³ $< \beta < 3000$ m²/m³. Many times, the terms compact-heat exchanger (CHE) and plate-heat-exchanger (PHE) are used indistinguishably.
- Meso heat exchangers if $0.2 < D_h/\text{mm} < 1$, or 3000 m²/m³ $< \beta < 10\,000$ m²/m³.
- Micro heat exchangers if $D_h < 0.2$ mm (or $\beta > 10\,000$ m²/m³). Human lung alveoli are typically 0.2 mm in size and have some 15 000 m²/m³.

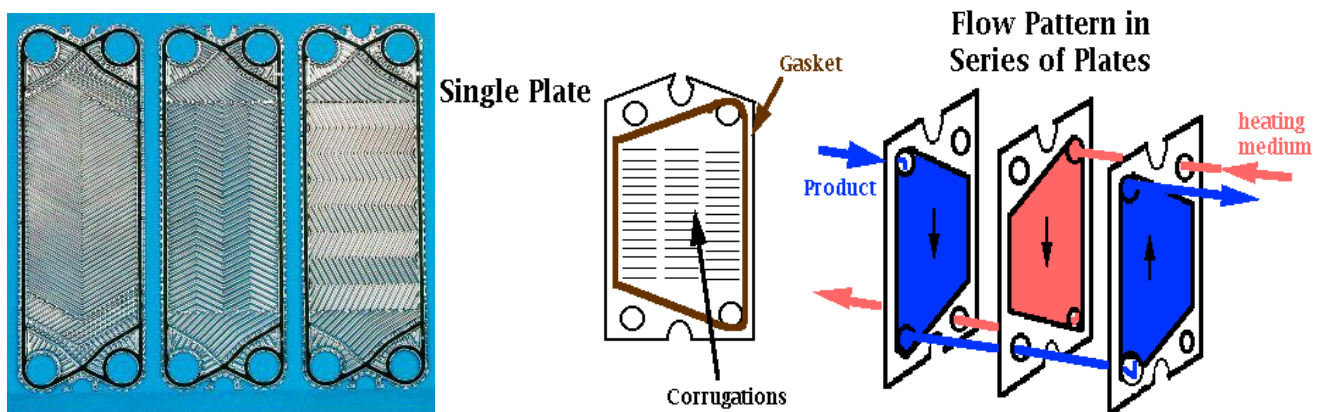
Heat exchangers are used to promote thermal energy flows at intermediate stages in process engineering, or as a final heat release to the environment, ambient air in most cases, which renders non-contact devices (as STHE and PHE) rather inefficient and recourse is to be made of contact heat exchangers as the wet cooling towers treated aside. A special case is that of marine engineering, where seawater is plenty available in the environment, greatly alleviating the thermal problem for heat-exchangers, but at a cost in materials compatibility (cupro-nickel or titanium must be used instead of copper or aluminium), since seawater is very corrosive and plenty of microorganisms. In order to mitigate the effects of seawater on heat exchangers, and to minimise hull-pass-throughs, only one central heat exchanger is cooled by seawater (a PHE usually), and all other required heat exchangers use clean fresh-water as an intermediate fluid loop to finally discharge the energy at the seawater exchanger (centralised cooling system); different fluid loop layouts can be used, normally grouping several thermal loads by proximity of location and by temperature level. For the latter, two levels are considered: high-temperature level (HT-circuit), say at >50 °C like for engine cooling circuits (main engine and auxiliaries), and low-temperature level (LT-circuit), say at <50 °C like for engine-oil-lubrication cooling, air-conditioners and refrigerators, electronic equipment, and so on; instead connecting the HT-circuit to the LT-circuit by means of a heat exchanger, it is better to use a partial mixing of the streams (regulated by a thermostatic valve). The standard design value for final heat release in ships is a seawater temperature of 32 °C, to allow for operation in all seas, in spite of fact that the largest share of operating time for most ships takes place in seas at 15 °C to 20 °C (initial cost cannot be avoided, but operation costs can be minimised by adjusting the seawater flow).

Heat exchangers are widely used in process control to promote or quench chemical reactions (by heating or cooling, respectively). The food industry makes use of heating to kill pathogen microorganisms (sterilisation), either after canning, or before packaging; the latter is most conveniently made for liquid stuff in heat exchangers. Sterilisation, i.e. the inactivation of all microorganisms, requires high-temperature processing, typically at 120 °C or more (i.e. under pressure, for aqueous stuff); to kill even the most resistant spores. In the pasteurisation process, however, a quick heating to 60 °C or 70 °C is applied to kill most bacteria without protein denaturing, but other microorganisms remain, what implies that quick cooling after pasteurisation is required (what makes heat pumps so convenient), and that vacuum or refrigeration is needed afterwards. The time-for-pasteurisation (or for sterilisation) depends on the microorganisms and the holding temperature; tabulated values are usually given for a processing temperature of $T_0=120$ °C, and can be extrapolated to other temperatures with the logarithmic law $\ln(t/t_0)=-m(T-T_0)/T_0$, with $m\approx 100$.

Plate heat exchangers

A plate heat exchanger, PHE, is a compact heat exchanger where thin corrugated plates (some 0.5 mm thick, bended 1 or 2 mm) are stacked in contact with each other, and the two fluids made to flow separately along adjacent channels in the corrugation (Fig. 1b). The closure of the staked plates may be by clamped gaskets, brazing (usually copper-brazed stainless steel), or welding (stainless steel, copper, titanium), the most common type being the first, for ease of inspection and cleaning. Additionally, a frame (end-plates and fixing rods) secures together the plate stack and connectors (sometimes PFHE, standing for plate-and-frame heat exchanger, is used instead of PHE).

Plate assembly is sketched in Fig. 2. Suitable channels, sometimes helped by the gaskets, control the flow of the two fluids, and allow parallel flow or cross flow, in any desired number of passes, one pass being most used. They have large conductance coefficients (up to $K=6000$ W/(m²·K) for liquid-to-liquid use), are ideally suited for low-viscosity fluids, the number of plates can be adjusted to the needs, and the transfer surface accessible to cleaning (the latter two advantages only for gasket assemblies; in any case, the gaskets should be changed if dismantled). The projected area of plates is usually taken as nominal heat transfer area, in spite of the real curved surfaces and lost space in gaskets and ports.



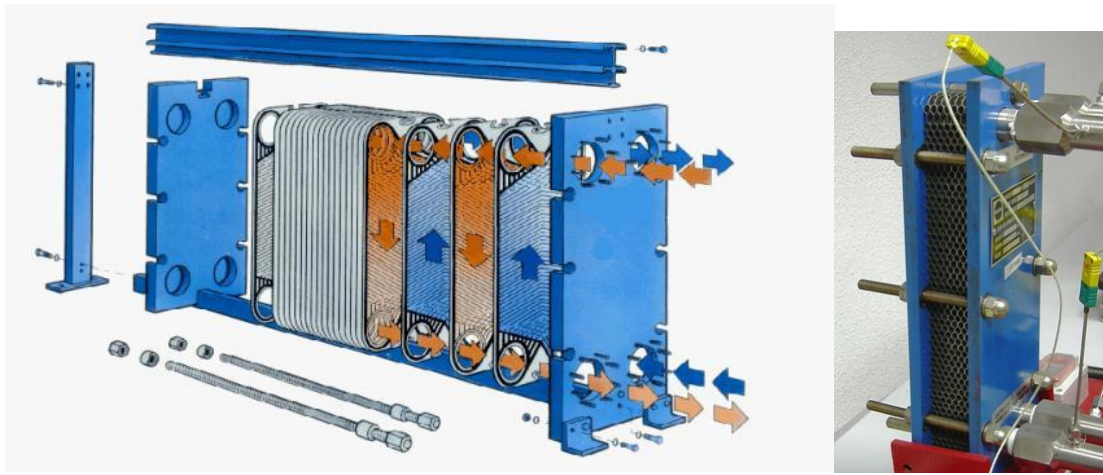


Fig. 2. Plate heat exchanger (single plates, exploded assembly, assembled).

Major limitations in PHEs are: maximum allowed pressure (usually below 1 MPa, although there are designs with 4 MPa), temperature range (usually limited to 150 °C by the gasket material, although there are designs allowing 400 °C), and price (but brazed PHEs are about half price of serviceable PHEs). Although typical PHE application is in liquid-to-liquid heat-transfer, special plate designs have been developed for phase-change applications. Higher working pressures and still good thermal performance can be achieved with hybrid plate-shell heat exchangers, where a plate stack is welded inside a shell (i.e. a kind of STHE with plates instead of tubes).

The PHE was developed in the 1920s in the food industry (for the pasteurization of milk), but they are taking over all markets now because of its compactness and efficiency (3 to 10 times more than STHE). They are used for process heating, cooling, in all cryogenic applications, and as an intermediate step in domestic water heaters, where consumable hot water (hot tap water) is produced in an intermediate heat exchanger from closed-loop fuel-fired hot water, to minimise solid depositions. PHE are often named CHE (compact heat exchangers), although the word compact can be added to any type of heat or mass transfer unit with specific area $>10^3 \text{ m}^2/\text{m}^3$.

Shell-and-tube heat exchangers

The shell-and-tube heat exchanger (STHE) is the most common type of heat exchanger. It originated from the jacketed-coil distiller, and is used in heavy industries (steam condensers, boilers), and residential hot-water and heating systems (fire-tube water heaters). Several details of STHes are presented in Fig. 3.



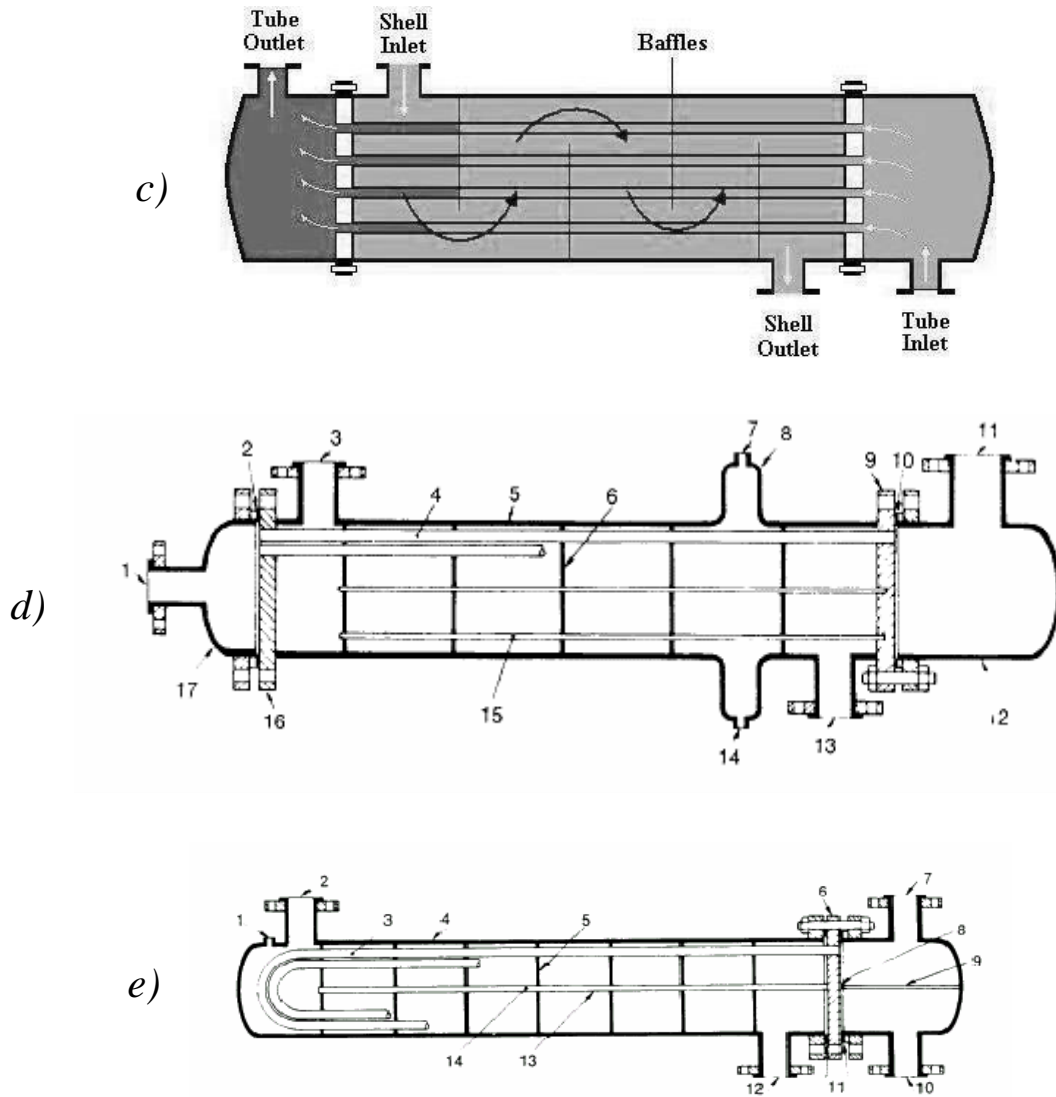


Fig. 3. Shell-and-tube heat exchangers (STHE): *a*) Small mounted STHE (in copper); *b*) Cut-out of a similar STHE; *c*) Schematic diagram of a STHX with one shell pass and one tube pass; *d*) Construction details with one shell pass and one tube pass (legend: 1, Tube-fluid exit; 2, Gasket; 3, Shell-fluid entrance; 4, Tubes; 5, Shell; 6, Baffles; 7, Purge; 8, Expansion diaphragm; 9, Tube holding plate; 10, Gasket; 11, Tube-fluid entrance; 12, Head cover; 13, Shell-fluid exit; 14, Drain; 15, Fastener; 16, Tube holding plate; 17, Head cover); *e*) Construction details of a STHX with two tube-passes and one shell-pass.

As can be seen in Fig. 3, each fluid can flow along several passes. In the tube-side, every set of tubes that the fluid travels through, before it makes a turn, is considered a pass. In the shell-side, a pass accounts for each main flow direction change. A standard code of practice in heat exchanger design and operation is that of TEMA (Tubular Exchangers Manufacturers Association). Other important source of standards are ASME (American Society of Mechanical Engineers; Boiler and Pressure Vessel Code), and, for heating and cooling loads, ASHRAE (American Society of Heating, Refrigerating and Air Conditioning Engineers).

Boilers

Basically, a boiler is a closed vessel or arrangement of enclosed tubes in which water is heated to supply steam (to drive an engine or turbine, or to provide heat); when other liquid than water is used, the boiler is more often named vaporiser (or evaporator). A second meaning of boiler is a domestic device burning

solid fuel, gas, or oil, to provide hot water, especially for central heating (better called a heater). Closely related to boilers are pressure cookers, i.e. strong hermetically sealed pots in which food may be cooked quickly under pressure at a temperature above the normal boiling point of water (in this case the intention is not to supply steam but to generate it for pressurising; the higher the pressure, the higher the boiling temperature).

Most boilers are fuel-fired, thus, they can be viewed as shell-and-tube heat exchangers (Fig. 4), where the hot fluid is the burnt gases, and the cold fluid the water stream. Heat transfer by radiation is important in boilers (and in furnaces) because of the high temperatures (some 2000 K). In most boilers, the air for combustion is previously heated by the exhaust gases in the stack. Typical efficiencies, measured as water enthalpy change divided by the combustion enthalpy (most often based on the standard low heating value of the fuel), are around 100% in modern condensation boilers (where part of the water vapour dissolved in the flue gases is condensed), around 90% for large non-condensing boilers, and around 80% for modern small non-condensing boilers. A boiler is often the largest energy consumer both at domestic and at industrial level, thus, great savings may be obtained by their proper selection, operation and maintenance.

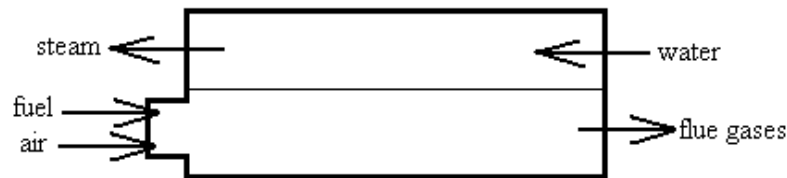


Fig. 4. A boiler is basically a burner and a heat exchanger.

Boiler development is linked to the steam-engine development, the driver of the Industrial Revolution. Hero of Alexandria, around 150 b.C., was the first to use a boiler to produce motion (a freely-spinning hollow-sphere boiler, exhausting through two hollow tubes oppositely bended), but it was Danis Papin, in 1679, the first to develop a pressure vessel and the first safety valve. Improvements were added by T. Savery in 1698, T. Newcomen in 1705, and J. Watt in 1785. Boilers were basically large kettles with a fire underneath, until Richard Trevithick, in 1804, implemented the first fire-tube boiler (a single fire-tube within the water tank), and made the first steam-locomotive. The Stephenson (father and son) greatly improved steam locomotives (with boilers working up to 0.5 MPa), and S. Wilcox in 1856 (G.H. Babcock joined Wilcox in 1866, creating one of the biggest boiler firms), developed the water-tube boiler, inherently safer because the high steam pressure only acts in the tubes (small), and not in the much larger shells. Rudimentary water-tube boilers were built before Wilcox's (a patent by Blakey dates 1766) but were inefficient and not much used. There were many boiler explosions in the XIX c. and early XX c. until the first ASME Code for Pressure Vessels of 1915 was widely adopted.

Large steam-flow-rates are needed in many process industries. On board a ship, steam may be required for:

- Propulsion of steam-turbine vessels: all nuclear vessels, and nearly all LNG carriers, are powered by steam turbines requiring high-pressure (e.g. 10 MPa), high-temperature (e.g. 800 K) steam. For other steam applications, any positive-pressure saturated-steam may work, with 0.7

MPa being common (up to 2 MPa in large heavy-fuel carriers). Boiler energy efficiency is in the range 0.75..0.9, increasing with size.

- Heating the load. For semi-solid substances: asphalt (>100 °C), heavy-fuel-oil (>50 °C), molasses (>50 °C), etc.
- Heating its own fuel. For day-service store of marine fuel, and for main and day-service stores of heavy fuels. Fuel heating used in all diesel ships with more than 10 MW engines.
- Heating the oil for the engine lubrication ('lub-oil'). Also for heating lub-oil/fuel mixtures for stowage or separation (bilge mud).
- Space heating for crew and passenger.
- Cleaning steam for inert spaces (hold compartments, engine room), and for habitable spaces (kitchens, laundry, lavatories).

Characteristics

Boiler specification is made in terms of several variables, among which the most important are:

- Capacity (mass flow rate production). Most common boilers are in the range from 2 kg/s to 10 kg/s of steam, supplied at typically 0.5 MPa to 5 MPa.
- Pressure level:
 - High pressure boiler (HPB, $\Delta p > 0.1$ MPa), that may be Power-HPB (if $D > 0.4$ m) or Miniature-HPB (if $D < 0.4$ m).
 - Low pressure boiler (LPB, $\Delta p < 0.1$ MPa). Sometimes, high pressure water-heaters (with $\Delta p > 1$ MPa) are considered as steam-LPB.
- Fuel used (gas, oil, coal, nuclear, electrical).
- Water flow arrangement (water must always be pressurised to get high-temperature steam):
 - Pressure-vessel (Watt-1785) with water externally heated by the flue gases.
 - Pyro-tubular (Scotch boiler). The flue gases penetrate or are created inside the pressure vessel with water. Evans-1800. $p_v < 2$ MPa. Tubes work at compression, and shell at expansion.
 - Aqua-tubular (drum boiler, Parsons-1894-TV-'Turbinia', 1.2 MPa, 1.5 MW). A non-pressurised vessel lodges the flame and flue gases, inside which pressurised pipes carry the water and/or its vapour. Tubes work at expansion, and shell does not work.
- Cost (first cost, running cost). Etc.

In a fire-tube boiler (Fig. 5), hot flue gases from the burner are channelled through tubes that are surrounded by the fluid to be heated. The body of the boiler is the pressure vessel and contains the fluid. In most cases this fluid is water that will be circulated for heating purposes or converted to steam for process use. Fire-tube boilers are relatively inexpensive, easy to clean, and more compact than water-tube boilers (although of smaller steam capacities, and not suitable for high pressure applications (up to 2 MPa only)).

Scotch "Fire Tube" Boiler Cross section and front views

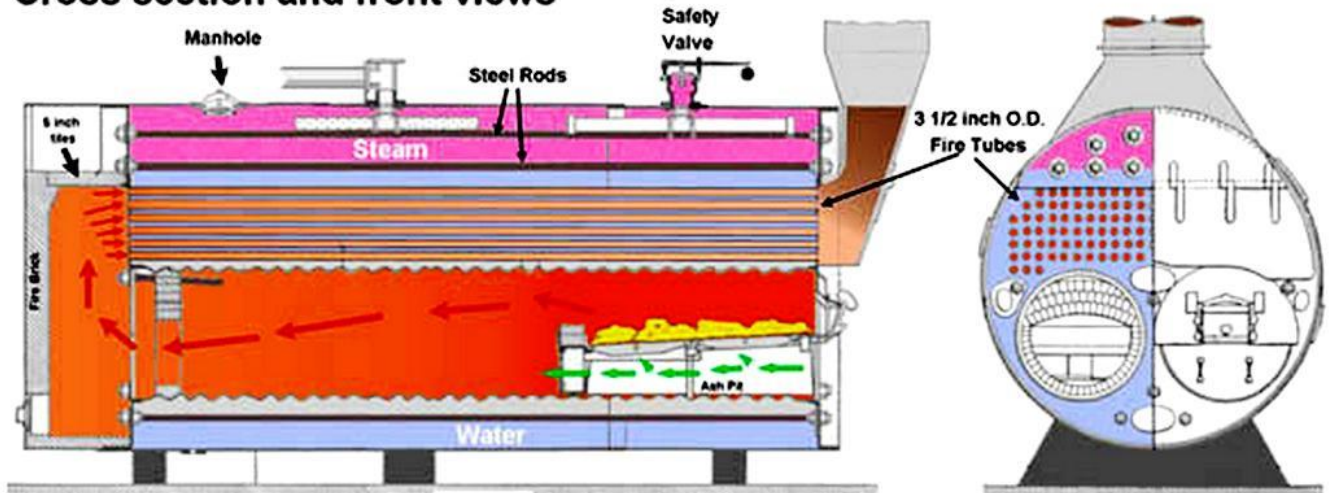


Fig. 5. A fire-tube boiler as those used in steam locomotives (Steamboat.com).

In a water-tube boiler, water flows through the tubes within a furnace in which the burner fires into. The tubes are connected to a steam drum on top and a mud drum at the bottom. Water-tube boilers typically produce steam or hot water for large industrial applications (less frequently for heating applications). They support higher pressure (up to 35 MPa) and temperature (900 K) than fire-tube boilers, but are more complex, larger (up to 50 m high, up to 60 kg/s of steam), and more expensive than fire-tube boilers. In supercritical boilers, water is heated at more than 22 MPa and converted to supercritical steam without any phase change.

Boiler design must consider not only the boiler itself (the heat exchanger where water becomes steam), but the steam system (heaters, piping, controls), the feed-water system (water conditioning, venting, purging), the fuel system (fuel/air ratio, piping), and the exhaust system (draught). Boiler material may be copper, steel, or cast iron. Contrary to what might be believed, boiler water is an expensive item because it must be de-mineralised (from typical 500 ppm total dissolved solids from the mains, down to 10 ppm), and de-gasified (CO_2 , O_2 and N_2 ; e.g. air-saturated water may have 10 mg/kg of dissolved oxygen, and it must be reduced to some 10 $\mu\text{g}/\text{kg}$). The latter is achieved in a device called deaerator, where feed-water to the boiler is heated and agitated by steam bubbling up to its vapour-equilibrium temperature at the working pressure of the deaerator (usually slightly above atmospheric pressure); non-condensable gases (and some entrained steam) are released through a vent valve.

Boiler safety is an important issue, since steam pressurised vessels have a lot of exergy and their explosion is very violent (many casualties have occurred). As a minimum, two independent, stepped safety devices are implemented; they open fully when needed. Two types of safety valves can be distinguished:

- Reversible safety valves (will close after low-pressure recovery; e.g. spring-loaded type). May be tested.
- Irreversible safety valves (for abnormal operation; permanent damage; must be replaced after use). Statistical test.

Besides safety devices, some pressure regulation device may be required for use or added for safety. Contrary to safety devices, these relief valves open proportionally to excess pressure.

Heat exchanger analysis

The thermal analysis of a heat exchanger is based on the simple coaxial configuration (Fig. 6), where one fluid goes along a pipe, and the other fluid goes along the annular section within a larger cylindrical sheath with openings at the ends, and in particular, to the counter-current configuration shown in Fig. 6a, more thermally-efficient than the co-flow set-up of Fig. 6b. The coaxial cross-section is sketched in Fig. 6c. Longitudinal temperature-profiles for the counter-flow and the co-flow configurations are also shown, and a detail of the transversal temperature profile across the wall separating the two fluids (Fig. 6d). The minimum temperature jump from one fluid to the other is called the (temperature) ‘approach’ of the heat exchanger.

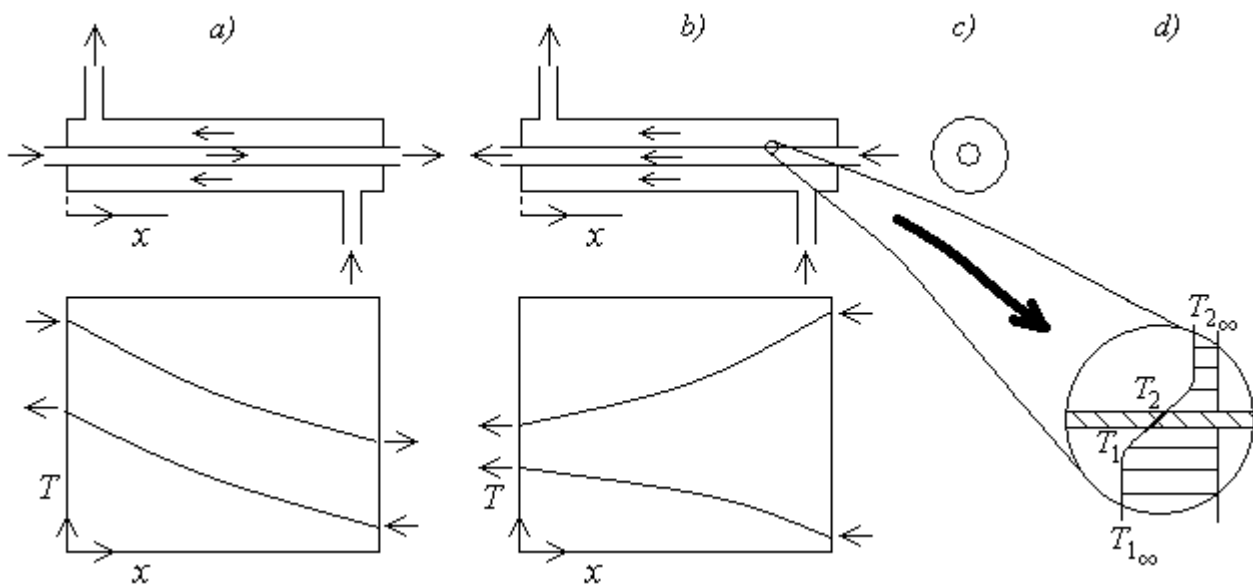


Fig. 6. Simple annular heat exchangers: a) sketch and temperature profile in a counter-flow configuration, b) sketch and temperature profile in co-flow, c) cross-section sketch, d) detail across the separating surface.

Heat transfer from one fluid to the other is a combined convection-conduction-convection process across the separating surface, as detailed in Fig. 2d, where the hotter fluid is subscripted '1' and the colder one '2'. The fact that the colder flow surrounds the hotter one in Fig. 6 is irrelevant to the results here developed, although it may be advantageous in practice to minimise heat-losses to the ambient (if its temperature difference to the ambient is lower than that of the hotter one), here neglected.

Overall heat-transfer coefficient

At steady state, the local heat flux through the wall separating the two fluids, from hotter to colder (Fig. 6c), is:

$$d\dot{Q} = h_1 dA_1 (T_{1\infty} - T_1) = k_s dA_m \frac{T_1 - T_2}{L_s} = h_2 dA_2 (T_2 - T_{2\infty}) \equiv K dA (T_{1\infty} - T_{2\infty}) \quad (2)$$

where the latter identity serves to introduce the local overall heat-transfer coefficient, K (traditionally named also U), associated to the actual choice of transfer area dA (either the internal one, $dA=dA_1$, the external one, $dA=dA_2$, or an appropriate average $dA=dA_m$, although in most circumstances the separating solid is so thin that all areas are practically the same, $dA=dA_1=dA_2=dA_m$).

If one introduces additional transfer factors to account for possible extended surfaces (e.g. fins or grooves) in terms of fin efficiency associated to root area, η_{fin} ([defined aside](#)), and possible degradations in the convective coefficients due to fouling, measured by an additional fouling resistance, R_f (often named 'fouling factor', although in that case should be treated as another efficiency term like the η_{fin} above), to be explained below, one gets for the overall heat-transfer coefficient, K :

$$K = \frac{1}{\frac{1}{\eta_{fin1}h_1} + R_{f1} + \frac{L_s}{k_s} + \frac{1}{\eta_{fin2}h_2} + R_{f2}} \quad (3)$$

To a first approximation, the conductive thermal resistance, L/k in (3), can always be neglected (except in low efficiency cases like distillers made of glass), and the convective resistance of liquids can also be neglected against that of gases. Table 1 below presents some typical K -values.

In practice, however, what matters is the average overall heat-transfer coefficient for the whole heat exchanger, K , defined in (3), instead of its local value defined by (2); we are using the same symbol for both the local and the global value, and some ambiguity also in temperature naming, since from now on T_1 and T_2 are going to be the bulk temperatures of the flows at a certain cross-section along the heat exchanger, i.e. what was named $T_{1\infty}$ and $T_{2\infty}$ in Fig. 6c, instead of the temperatures at the wall.

Notice, by the way, that the overall heat-transfer coefficient, K , is also very convenient in the study of heat transfer between two fluids by natural convection, like for windows and radiators, where typical values are $K=1$ W/(m²·K) for a double-glass window, $K=3$ W/(m²·K) for a single-glass window, $K=6$ W/(m²·K) for a planar solar collector, or $K=10$ W/(m²·K) for a heating 'radiator'.

In the case of plate heat exchangers, the overall heat-transfer coefficient, K , is poorly defined because only the product KA enters into the equations, and the contact area, A , between the two fluids is difficult to define and measure due to the corrugations.

Fouling

One generic problem of heat transfer is the effect of surface contamination on heat-transfer rates, that may be due for instance to chemical attack at the interface between solids in heat conduction, crust build-up at walls in heat convection flows, or dust deposition in heat radiation surfaces. Fouling (i.e. dirt and depositions) is detrimental within heat exchangers because it adds a thermal resistance to heat, it adds a fluid-dynamic resistance to flow, and it is difficult (sometimes impossible) to clean.

Typical fouling factors used in heat transfer analysis are $R_f=0.002$ m²·K/W for fuel oil, $R_f=0.001$ m²·K/W for flue gases, $R_f=0.0004$ m²·K/W for air and refrigerant vapours, $R_f=0.0002$ m²·K/W for normal liquids,

and $R_f=0.0001 \text{ m}^2 \cdot \text{K}/\text{W}$ for steam and for water (or sea water) below $50 \text{ }^\circ\text{C}$. If one fluid is a gas, fouling has a negligible effect on transfer rates.

Fouling is typically due to algae growth on cold surfaces, to salt deposition on hot surfaces, and to unfiltered dirt clogging. All industrial circuits cooled with natural fresh or sea water, are subjected to biological fouling (a bio-film settlements of living organisms). Although physical screening, physical cleaning and chemical dosing can manage to get a long trouble-free operation, nowadays environmental restrictions may impinge on that; e.g. chemical dosing with chlorine concentrations up to $10 \text{ mg}/\text{L}$ were common in the past, but now chlorine discharge to the water environment have been severely restricted.

Heat exchanger modelling

Engineering practice makes use of off-the-shelf heat exchangers (PHE or STHE), but thermal performance and design are best explained in terms of the simple parallel-flow configurations shown in Fig. 6 above, and in particular for the counter-flow case in Fig. 6a.

For a differential slice of length dx along the counter-current heat-exchanger in Fig. 2a, the longitudinal energy balance for each fluid, and the transversal energy balance across the separating wall, assuming the hot fluid is subscripted '1', are:

$$d\dot{Q} = -\dot{m}_1 c_1 dT_1 = -\dot{m}_2 c_2 dT_2 = K dA (T_1 - T_2) \quad (4)$$

where $d\dot{Q}$ is, as in (2), the transversal heat flow from hot (T_1) to cold (T_2) flows across a differential annulus of length dx , $\dot{m}_1 c_1$ and $\dot{m}_2 c_2$ the thermal-capacity rates of each stream (the perfect-substance model is assumed to ease writing, but phase changes would be equally treated in terms of enthalpies instead of temperatures), and dA the wetted annular are of length dx (either internal, external, or some average area) The negative signs associated to dT in (4) come from the negative slopes of the temperature profiles in Fig. 6a.

Integrating (4) from one end ($x=0$) to the other end ($x=L$) along the heat exchanger (Fig. 6a) yields:

$$\dot{Q} = \dot{m}_1 c_1 (T_{1,\text{in}} - T_{1,\text{out}}) = \dot{m}_2 c_2 (T_{2,\text{out}} - T_{2,\text{in}}) = KA \Delta T_{12,\text{mean}} \quad (5)$$

where $\Delta T_{12,\text{mean}}$ is an appropriate average, defined by (5) in terms of the overall heat-transfer coefficient K and the choice of wet area A . This average temperature approach can be obtained by integration of a combination of the three equations in (4), and ulterior comparison with (5); i.e., from (4):

$$d\dot{Q} = \frac{-dT_1}{\dot{m}_1 c_1} = \frac{-dT_2}{\dot{m}_2 c_2} = -\frac{dT_1 - dT_2}{\frac{1}{\dot{m}_1 c_1} - \frac{1}{\dot{m}_2 c_2}} = K dA (T_1 - T_2) \quad (6)$$

and integrating the latter equality from $x=0$ to $x=L$:

$$\ln \frac{(T_1 - T_2)_{x=L}}{(T_1 - T_2)_{x=0}} = -KA \left(\frac{1}{\dot{m}_1 c_1} - \frac{1}{\dot{m}_2 c_2} \right) = KA \left(\frac{T_{1,\text{out}} - T_{1,\text{in}}}{\dot{Q}} - \frac{T_{2,\text{in}} - T_{2,\text{out}}}{\dot{Q}} \right) = \frac{(T_1 - T_2)_{x=L} - (T_1 - T_2)_{x=0}}{\Delta T_{12,\text{mean}}} \quad (7)$$

which finally yields:

$$\Delta T_{12,\text{mean}} = \frac{(T_1 - T_2)_{x=L} - (T_1 - T_2)_{x=0}}{\ln \frac{(T_1 - T_2)_{x=L}}{(T_1 - T_2)_{x=0}}} \equiv \Delta T_{\text{LMTD}} \quad (8)$$

what shows that, for the counter-flow configuration in Fig. 6a, with constant-thermal-capacity fluids, the appropriate mean-value of the temperature jump across the separating wall, is the logarithmic mean temperature difference (LMTD) defined by (8), which nearly coincides with the arithmetic mean when the approach at both ends is of the same order (the singularity for this case in (8) can easily be circumvented).

In an analogous manner, it can be demonstrated that the LMTD also applies to the coaxial heat exchanger in co-flow shown in Fig. 6b, but not in a more general case of multi-pass or cross-flow heat exchangers, neither to the case of highly-varying thermal capacities (as in two-phase mixtures); in those cases, an empirical correction factor, F , can be defined to get an appropriate average approach, $\Delta T_{12,\text{mean}} = F \Delta T_{12,\text{LMTD}}$, in terms of the LMTD defined in (8).

Thus, the thermal modelling of a heat exchanger reduces to the three equations (5) with the eight variables: \dot{Q} , $\dot{m}_1 c_1$, $T_{1,\text{out}}$, $T_{1,\text{in}}$, $\dot{m}_2 c_2$, $T_{1,\text{out}}$, $T_{1,\text{out}}$, and KA . Notice that K and A do not appear separately (as for \dot{m} and c). Although a brute accounting would yield $8 \cdot 7 \cdot 6 / (3 \cdot 2) = 56$ possible combinations for the trio of solvable unknowns, in practice there are only a few interesting cases, namely:

- Testing problem. This is the case where an actual heat exchanger is bench-tested by measuring all input and output flow variables: $\dot{m}_1 c_1$, $T_{1,\text{out}}$, $T_{1,\text{in}}$, $\dot{m}_2 c_2$, $T_{1,\text{out}}$, and $T_{1,\text{out}}$. We have the three equations (5) but only two unknowns \dot{Q} , and KA , meaning that there is one redundancy in the measurement (what might be used to check that the overall heat losses were really negligible). As in general, only the product KA can be computed, but it is not difficult to get an estimate of A from the external dimensions and the knowledge of the number of plates in a PHE or the number of tubes in a STHE. Conversely, K might be estimated by heat-convection correlations in terms of the type of fluids, their velocities, and the geometry of flow, or even based on typical values for the overall coefficient (e.g. $K \approx 2000 \text{ W}/(\text{m}^2 \cdot \text{K})$ for water-to-water heat exchangers).
- Performance: problem. Given the entry conditions for the two flows ($\dot{m}_1 c_1, T_{1,\text{in}}$, $\dot{m}_2 c_2$, and $T_{2,\text{in}}$), and the size (measured by the transfer area, A), and assuming the overall heat-transfer coefficient known, K , determine the heat rate exchanged, \dot{Q} , and the output temperatures ($T_{1,\text{out}}$ and $T_{2,\text{out}}$), i.e. three unknowns for the three equations in (4), if the LMTD approach is applied (otherwise, see the NTU approach below).
- Design problem. Given the entry conditions for the two flows ($\dot{m}_1 c_1, T_{1,\text{in}}$, $\dot{m}_2 c_2$, and $T_{2,\text{in}}$), and the heat rate exchanged, \dot{Q} , and assuming the overall heat-transfer coefficient known (K), determine the size (measured by the transfer area, A) and the outlet conditions: i.e. three unknowns for the three equations in (5). The step-by-step procedure to solve the design problem is:

1. Establish the governing equations (5) and perform the analysis of data and unknowns.
2. Compute the outlet temperatures from the flow-rate capacities and heat load.
3. Estimate the required transfer area using typical values of overall heat-transfer coefficients (e.g. Table 1) and the ΔT_{LMTD} from (8), or other averaged approach. This order of magnitude may be good enough for non-critical designs, and will help to check more refined analysis, as following.
4. Select a type of heat exchanger based on type of fluids and expected loading conditions.
5. In the case of STHE, select which fluid goes on the tube-side: either the dirtiest one (because shells cannot be cleaned), or the high-pressure one (because small tubes resist pressure better than large shells). In the case of other types of heat exchangers, do it in a similar way.
6. Select tube diameter, d , according to available tube sizes.
7. Estimate the required flow cross-section, $A = \dot{m}/(\rho v)$, assuming a typical speed of $v=1$ m/s for liquids, or $v=10$ m/s for gases.
8. Estimate the number of tubes, N , from $A = \dot{m}/(\rho v) = N\pi dL$ assuming a typical Reynolds number, $Re=vd/\nu$, of 5000.
9. Find if several tube-passes are needed by estimating the shell diameter, D , by $D = 2d\sqrt{N}$, and checking that the resulting overall slenderness is around $L/D \approx 6$.
10. Estimate pressure losses in the tube side (drag on pipe flow can be found in [Forced and Natural Convection](#), but other losses in bends and fittings may be relevant too).
11. Consider that the uncertainty in the above procedure may be $\pm 50\%$, and guess if a more refined design is needed at that stage (in that case, one must resort to more specific literature, or to experimentation, as a last recourse).

Table 1. Order of magnitude values of global or overall heat-transfer coefficients, K , for heat exchangers.

Configuration	Typical value K [W/(m ² ·K)]	Typical range K [W/(m ² ·K)]
Gas-to-gas heat exchanger at normal pressure	20	5..50
Gas-to-gas heat exchanger at high pressure	200	50..500
Liquid-to-gas or gas-to-liquid heat exchanger	50	10..100
Liquid-to-liquid tubular heat exchanger	1000	200..2000
Liquid-to-liquid plate heat exchanger	2500	500..5000
Condenser, to a gas	50	10..100
Condenser, to a liquid	3000	500..6000
Vaporiser, to a gas	50	10..100
Vaporiser, to a liquid	5000	500..10 000
Vaporiser, to a condensing gas	3000	600..6000

Exercise 1. A hot water stream of 0.2 kg/s at 80 °C is to be cooled in an annular counter-flow heat-exchanger with 0.5 kg/s of ambient water at 15 °C. The inner tube is made of stainless steel and has 20 mm internal diameter and 1 mm thickness, whereas the internal diameter of the outer tube has 30 mm in diameter. Find the convective coefficients, the overall heat-transfer coefficient, the heat flow-rate, and the exit temperatures.

Solution. A rough typical-value analysis yields $h=5000 \text{ W}/(\text{m}^2\cdot\text{K})$ from [Table 6 in Heat Convection](#), with a corresponding overall heat-transfer coefficient of $K=h/2=2500 \text{ W}/(\text{m}^2\cdot\text{K})$ from (3), having neglected conductive resistance across the separating tube wall, and all kind of fouling; notice, however, that the typical K -value suggested in Table 1 above for liquid-to-liquid tubular heat-exchanger is just $K=1000 \text{ W}/(\text{m}^2\cdot\text{K})$. With the latter, and taking the temperature difference at inlet points instead of the correct LMTD-value, the heat flow rate would be $\dot{Q} = KA\Delta T = 1000 \cdot 0.12 \cdot (80 - 15) = 7.8 \text{ kW}$, where the transfer area has been approximated by the internal one, $A = \pi DL = 3 \cdot 0.02 \cdot 2 = 0.12 \text{ m}^2$. From the energy balance of each stream, $\dot{Q} = \dot{m}c\Delta T$, we finally get the outlet temperatures: $T_{\text{hot,out}} = T_{\text{hot,in}} - \dot{Q}/(\dot{m}c) = 80 - 7.8/(0.2 \cdot 4.2) = 71 \text{ }^\circ\text{C}$, $T_{\text{cold,out}} = T_{\text{cold,in}} + \dot{Q}/(\dot{m}c) = 15 + 7.8/(0.5 \cdot 4.2) = 19 \text{ }^\circ\text{C}$.

But the expected uncertainty in the h -values and K -values assumed above, some 50% or more, are only acceptable for an order-of-magnitude analysis. A more refined analysis requires a more precise determination of the convective coefficients h , what is done through the empirical correlations presented under [Forced and natural convection, aside](#). Assuming the hot stream runs along the inner tube (the typical case), its bulk speed is $u=0.64 \text{ m/s}$, $Re=12\,700$, $Pr=2.7$ (evaluated at $80 \text{ }^\circ\text{C}$), and using Dittus-Boelter correlation for turbulent flows ($12700 > 2300$), $Nu=0.023Re^{0.8}Pr^{0.3}=60$, and $h=kNu/D=1800 \text{ W}/(\text{m}^2\cdot\text{K})$. For the cold stream, the hydraulic diameter must be used, $D_h=4A/p=D_2-D_1=30-21=9 \text{ mm}$, what yields a bulk speed of $u=1.53 \text{ m/s}$, $Re=12\,200$, $Pr=7.7$ (evaluated at $15 \text{ }^\circ\text{C}$), $Nu=0.023Re^{0.8}Pr^{0.3}=97$, and $h=kNu/D=7260 \text{ W}/(\text{m}^2\cdot\text{K})$. The proper K -value is now $K=1/(1/h_1+\delta/k+1/h_2)=1300 \text{ W}/(\text{m}^2\cdot\text{K})$, and the heat flow-rate taking the temperature difference at inlet points instead of the correct LMTD-value, $\dot{Q}=10.7 \text{ kW}$, what yields $T_{\text{hot,out}}=67 \text{ }^\circ\text{C}$ and $T_{\text{cold,out}}=20 \text{ }^\circ\text{C}$. With these values, we can compute a better approximation to the logarithmic mean temperature difference using (8), $\Delta T_{\text{LMTD}}=56 \text{ }^\circ\text{C}$ (instead of the $80-15=65 \text{ }^\circ\text{C}$ former approximation), and new, more accurate values for $\dot{Q}=9.2 \text{ kW}$, $T_{\text{hot,out}}=69 \text{ }^\circ\text{C}$ and $T_{\text{cold,out}}=19 \text{ }^\circ\text{C}$. We could go on with further iterations, correcting the computed material properties at an average bulk temperature between inlet and outlet (instead of the assumed inlet values), and so on, but there is usually no need to better fit final figures when the initial data uncertainty is not well defined.

Non-dimensional method of heat-exchanger modelling: the NTU approach

The LMTD method above-explained offers a full model of a heat exchanger, with 8 variables linked by the three equations in (5) with the definition in (8) (supplemented, if needed, with the mentioned correction factor F to account for multi-pass or cross-flow configurations). However, solving the performance problem stated above, i.e. finding \dot{Q} , $T_{1,\text{out}}$ and $T_{2,\text{out}}$, given \dot{m}_1c_1 , $T_{1,\text{in}}$, \dot{m}_2c_2 , $T_{2,\text{in}}$, and KA , gives a non-linear system because of the LMTD-expression (8), involving tedious iterations, what can be circumvented by another approach, based on a non-dimensional parameter introduced by Kays-London in 1955, namely the number of transfer units (NTU), N , defined in terms of the minimum of the two thermal mass-flow-capacities, $(\dot{m}c)_{\text{min}}$, as:

$$N \equiv \frac{KA}{(\dot{m}c)_{\text{min}}} \quad (9)$$

Two other non-dimensional parameters complete the new model: the heat-exchanger effectiveness, η (sometimes denoted ε), and the heat capacity ratio, c , defined by:

$$\eta \equiv \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\dot{m}_1 c_1 (T_{1,\text{in}} - T_{1,\text{out}})}{(\dot{m}c)_{\min} (T_{1,\text{in}} - T_{2,\text{in}})} = \frac{\dot{m}_2 c_2 (T_{2,\text{out}} - T_{2,\text{in}})}{(\dot{m}c)_{\min} (T_{1,\text{in}} - T_{2,\text{in}})} = \frac{KA\Delta T_{12,\text{mean}}}{(\dot{m}c)_{\min} (T_{1,\text{in}} - T_{2,\text{in}})} \quad (10)$$

$$c \equiv \frac{(\dot{m}c)_{\min}}{(\dot{m}c)_{\max}} \quad (11)$$

The heat capacity ratio, c , ranges from $c=0$ in the case where one fluid changes of phase (vaporisers or condensers, because then the energy change in that fluid is without temperature change and thus its thermal capacity tends to infinity), to $c=1$ in the case where both flows have the same $\dot{m}c$ -value, as in most recuperative heat exchangers. When one fluid is liquid and the other gas, the $\dot{m}c$ -value for the gas stream is usually much lower than its counterpart for the liquid, and $c \rightarrow 0$.

The efficiency of a heat exchanger is the ratio of heat-flow actually exchanged, \dot{Q} , to the maximum heat-flow exchangeable for given input conditions if the length were infinity, \dot{Q}_{\max} .

With this notation, the three equations (5) that govern heat-exchanger modelling, can now be stated as (assuming that the flow subscripted '1' has the lowest thermal capacity):

$$\eta = \frac{T_{1,\text{in}} - T_{1,\text{out}}}{T_{1,\text{in}} - T_{2,\text{in}}}, \quad \eta = \frac{T_{2,\text{out}} - T_{2,\text{in}}}{c(T_{1,\text{in}} - T_{2,\text{in}})}, \quad \eta = N \frac{\Delta T_{12,\text{mean}}}{T_{1,\text{in}} - T_{2,\text{in}}} = \eta(N, c) \quad (12)$$

where the latter equation has been set in the generic functional form $\eta = \eta(N, c)$, instead of in terms of a suitable averaged approach (which was demonstrated to be the LMTD in the case of simple coaxial-tube heat exchangers, to be modified with the F factor for other configurations). We can now find the explicit form of $\eta = \eta(N, c)$ in the case of the counter-flow configuration in Fig. 6a, as done for the LMTD, namely, by

$$\begin{aligned} \ln \frac{(T_1 - T_2)_{x=L}}{(T_1 - T_2)_{x=0}} &= -KA \left(\frac{1}{\dot{m}_1 c_1} - \frac{1}{\dot{m}_2 c_2} \right) \rightarrow \ln \frac{T_{1,\text{out}} - T_{2,\text{in}}}{T_{1,\text{in}} - T_{2,\text{out}}} = -N(1-c) \Rightarrow \\ \rightarrow \ln \frac{T_{1,\text{out}} - T_{2,\text{in}}}{T_{1,\text{in}} - T_{2,\text{out}}} &= \ln \frac{(T_{1,\text{out}} - T_{1,\text{in}}) + (T_{1,\text{in}} - T_{2,\text{in}})}{(T_{1,\text{in}} - T_{2,\text{in}}) - (T_{2,\text{out}} - T_{2,\text{in}})} = \ln \frac{-\eta + 1}{1 - c\eta} = -N(1-c) \Rightarrow \\ \Rightarrow \eta(N, c) &= \frac{1 - e^{-N(1-c)}}{1 - ce^{-N(1-c)}} \end{aligned} \quad (13)$$

Thus, instead of the three equations (5) helped with (8) and an F -factor correction, now we have the three equations (12) helped with (13) and another correction as before for multi-pass or cross-flow configurations, which in this NTU-method can be established analytically in some cases as a particular $\eta(N, c)$ -functions other than (13), as here presented analytically in Table 2, or presented in graphical form in most Heat Transfer texts. Of course, instead of using special $\eta(N, c)$ -functions, the old correction factor, F , to be multiplied by the counter-flow $\eta(N, c)$ -functions (13) can be used, similarly as for the LMTD-approach.

Table 2. Effectiveness relations of some heat exchanger configurations.

Configuration	$\eta = \eta(N, c)$	$N = N(\eta, c)$
Coaxial pipes in counter-flow	$\eta = \frac{1 - e^{-N(1-c)}}{1 - ce^{-N(1-c)}}$	$N = \frac{1}{1-c} \ln \frac{1 - c\eta}{1 - \eta}$
Coaxial pipes in co-flow	$\eta = \frac{1 - e^{-N(1+c)}}{1 + c}$	$N = \frac{\ln[\eta(1+c) - 1]}{1+c}$
STHE with 1 shell-pass and any even tube-passes	$\eta = \frac{2}{1+c + \sqrt{1+c^2}} \frac{1 + e^{-N\sqrt{1+c^2}}}{1 - e^{-N\sqrt{1+c^2}}}$	$N = \frac{1}{\sqrt{1+c^2}} \ln \frac{\frac{2}{\eta} + \sqrt{1+c^2} - (1+c)}{\frac{2}{\eta} - \sqrt{1+c^2} - (1+c)}$
Cross-flow, single pass, $(\dot{m}c)_{\min}$ – mixed, $(\dot{m}c)_{\max}$ – unmixed	$\eta = 1 - e^{-\frac{\exp(-cN) - 1}{c}}$	$N = -\frac{\ln[c \ln(1-\eta) + 1]}{c}$
Cross-flow, single pass, $(\dot{m}c)_{\min}$ – unmixed, $(\dot{m}c)_{\max}$ – mixed	$\eta = \frac{1}{c} \left(1 - e^{1-c[1-\exp(-N)]} \right)$	$N = -\ln \left[1 + \frac{\ln(1-c\eta)}{c} \right]$
Any heat exchanger with $c=0$ (phase change)	$\eta = 1 - e^{-N}$	$N = -\ln(1-\eta)$

The variety of flow configurations in a heat exchanger may be very wide, as depicted in Fig. 7 for a non-standard STHE, where the temperature profiles have been sketched. And not only heat-flow correlations are needed, but friction correlations too, to know the pressure losses on each fluid. In actual practice, after a preliminary analysis using the models presented above, recourse is made to the particular data supplied by the manufacturer (once selected).

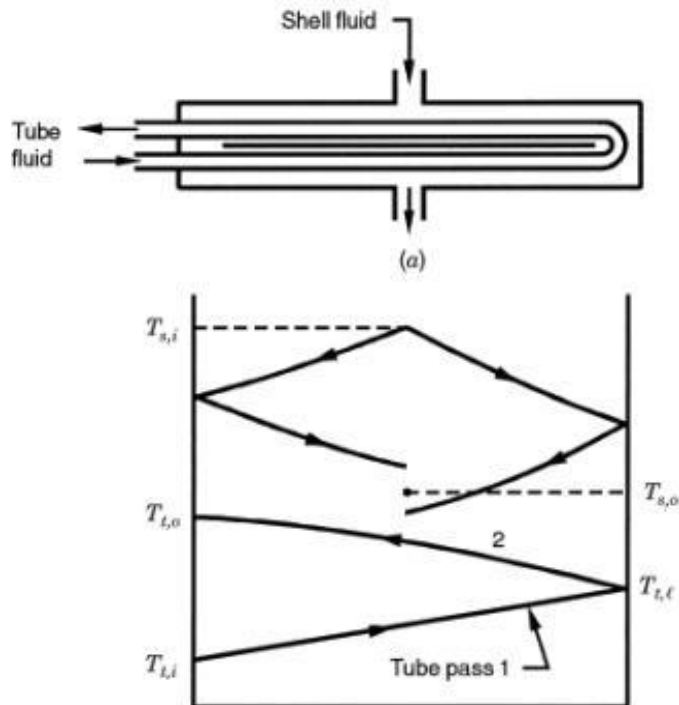


Fig. 7. Temperature profiles along a cross-flow shell-and-tube heat exchanger.

Preliminary sizing

It is important to have simple dimensioning procedures to allow for preliminary analysis of thermal systems with heat exchangers. For instance, when studying a new heat-pump application, one has to choose appropriately the operating temperatures, based on the heat source and heat sink temperatures

available; from these condenser and vaporiser temperatures selected, and the selected working fluid, one can solve the heat-pump cycle and compute the mass-flow and power requirements for the equipment, what serves for dimensioning the compressor and make a sound selection among the available offer. But if the fluid working temperatures are chosen too close to the heat source and heat sink values, very efficient and expensive heat exchangers will be required to achieve that goal, and if ample margin is let, there will be no problem to find a cheap heat exchanger to do the job, but at the expense of over-sizing the heat-pump compressor.

In conclusion, there is the omnipresent trade-off between saving on first cost at the expense of operation costs (by choosing cheap components with high consumption), and saving on operation costs at the expense of initial cost (by choosing more expensive components with lower consumption). But one cannot optimise everything at once, and engineers should not, because many times their results are not to be implemented in the final product but just iterative inputs to the design process, to be discarded after an overall view of the process dictates new changes for the next iteration. That is why the modelling effort (be it of a heat exchanger or of another device or process), should be a proportional share of the whole effort at a time. In our case, the recommendations, from simplest to more complex modelling of heat exchangers, are:

- Just account for a reasonable minimum temperature approach between the two fluids, e.g. $\Delta T_{12,\min}=5$ K (1 K approach might be enough with good plate heat exchangers for liquids, but 50 K may be required for gas-to-gas heat exchangers). At a later stage, use the LMTD-method or the NTU-method to find the required contact area of the heat exchanger, A , for a typical overall heat-transfer coefficient, K , guessed from Table 1.
- A better model may be to account for a reasonable heat-exchanger efficiency, e.g. $\eta=0.8$, and use (12) to find the outlet temperatures. At a later stage, use the LMTD-method or the NTU-method to find the required contact area of the heat exchanger.
- A more accurate model considers the system of equations ((5) or (12), plus specific relations for the F -factor or for $\eta(N,c)$ -function as in Table 2), and the unknowns as a separate procedural-module to be computer-solved in conjunction with the rest of the thermal problem.
- As the last resort of theoretical analysis, the overall heat-transfer coefficient, K , can be computed using (3) with the convective coefficients, h , at each side computed with appropriate [forced-convection correlations found aside](#), plus additional fouling data (see above) if necessary. Beyond this analysis, one may conduct detailed numerical simulation using a CFD package, and experimental testing.

A final remark is that we have just focused on the thermal analysis and design of heat exchangers, i.e. on their heat-transfer duty, trying to incorporate general economic criteria always present in any kind of engineering project, but, as said in the beginning, there are many other important aspects in heat exchanger analysis, as pressure-loss calculations for pumping needs, materials compatibility with the fluids, structural design, maintainability, cost...

[Back to Heat and mass transfer](#)

[Back to Thermodynamics](#)